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PHYSICS for you

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Managing Editor
Mahabir Singh
Editor
Anil Ahlawat

Corporate Office:
Plot 99, Sector 44 Institutional area, Gurugram -122 003 (HR).
Tel : 0124-6601200 e-mail : info@mtg.in website : www.mtg.in
Regd. Office:
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GET SET GO for NEET

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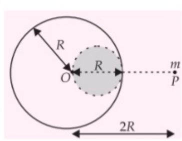
1. The density of a material in CGS system of units is 4 g cm^{-3}. In a system of units in which unit of length is 10 cm and unit of mass is 100 g, the value of density of material will be
(a) 0.4 (b) 40 (c) 400 (d) 0.04

2. A tuning fork vibrating with a sonometer having 20 cm wire produces 5 beats per second. The beat frequency does not change when the length of the wire is changed to 21 cm. The frequency of the tuning fork (in Hz) is
(a) 200 (b) 210 (c) 205 (d) 215

3. A uniform sphere of mass M and radius R exerts a force F on a small mass m situated at a distance of $2R$ from the centre O of the sphere. A spherical portion of diameter R is cut from the sphere as shown in figure. The force of attraction between the remaining part of the sphere and the mass m will be



(a) $\frac{F}{3}$ (b) $\frac{2F}{3}$ (c) $\frac{4F}{3}$ (d) $\frac{7F}{9}$

4. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is
(a) $I_Y = 32\, I_X$ (b) $I_Y = 16\, I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64\, I_X$

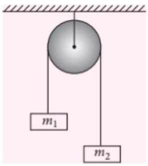
5. A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of 3 m s^{-2} for 0.5 minute. If the maximum height reached by it is 80 m, then the angle of projection is (Take g = 10 m s^{-2})
(a) $\tan^{-1}(3)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$
(c) $\tan^{-1}\left(\frac{4}{9}\right)$ (d) $\sin^{-1}\left(\frac{4}{9}\right)$

6. A metal ball of radius 10^{-4} m and density 10^4 kg m^{-3} falls freely under gravity through a distance h and enters a tank of water. It is found that after entering the water, the velocity of ball does not change. What is the value of h?
[η for water = 10^{-5} Pa, g = 10 m s^{-2} and $\rho_{water} = 10^3$ kg m^{-3}]
(a) 10 m (b) 15 m (c) 18 m (d) 20 m

7. The pressure of a gas in a 100 mL container is 200 kPa and the average translational kinetic energy of each gas particle is 6.0×10^{-21} J. Find the number of moles of gas in the container.
(Avogadro's number = 6×10^{23} mol^{-1})
(a) 8.3 (b) 8.3×10^{-2}
(c) 8.3×10^{-3} (d) 8.3×10^{-4}

8. Water from a tap emerges vertically downwards with an initial speed of 1.0 m s^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and the flow is steady and streamline, then the cross-sectional area of the stream, 0.15 m below the tap will be
(a) 2×10^{-5} m^2 (b) 3×10^{-5} m^2
(c) 4×10^{-5} m^2 (d) 5×10^{-5} m^2

9. Two masses m_1 = 1 kg and m_2 = 2 kg are connected by a light inextensible string and suspended by means of a weightless pulley as shown in the figure. Assuming that both the masses start from rest, the distance travelled by the centre of mass in 2 s is (Take g = 10 m s^{-2})



(a) $\frac{20}{9}$m (b) $\frac{40}{9}$m (c) $\frac{2}{3}$m (d) $\frac{1}{3}$m

10. A given mass of gas is compressed isothermally until its pressure is doubled. It is then allowed to expand adiabatically until its original volume is restored and its pressure is then found to be 0.75 of its initial pressure. The ratio of the specific heats of the gas is approximately
(a) 1.20 (b) 1.41 (c) 1.67 (d) 1.83

11. A particle starts from rest and travels a distance s with uniform acceleration, then it travels a distance 2 s with uniform speed. Finally it travels a distance 3 s with uniform retardation and comes to rest. If the complete motion of the particle is a straight line then the ratio of its average velocity to maximum velocity is
(a) 6/7 (b) 4/5 (c) 3/5 (d) 2/5

12. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the wire in the middle is of iron. What is the ratio of their diameters $\frac{D_{copper}}{D_{iron}}$ if each wire has the same tension ?
Given : $Y_{copper} = 1.1 \times 10^{11}$ Pa and $Y_{iron} = 1.9 \times 10^{11}$ Pa.
(a) 1.2 (b) 1.3 (c) 1.5 (d) 2.5

13. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_P/C_V of the mixture is
(a) 1.4 (b) 1.54 (c) 1.59 (d) 1.62

14. The potential energy of a 1 kg particle free to move along the x-axis is given by
$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) \text{J}$$
The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m s^{-1}) is
(a) 2 (b) $\frac{3}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

15. The moment of inertia of a uniform disc about an axis passing through its centre and perpendicular to its plane is 1 kg m^2. It is rotating with an angular velocity 100 rad s^{-1}. Another identical disc is gently placed on it so that their centres coincide. Now these two discs together continue to rotate about the same axis. Then the loss in kinetic energy in kJ is
(a) 2.5 (b) 3.0 (c) 3.5 (d) 4.0

SOLUTIONS

1. **(b) :** Dimensional formula of density,
$\rho = [M^1 L^{-3} T^0]$

CGS system	Given system
M_1 = 1 g	M_2 = 100 g
L_1 = 1 cm	L_2 = 10 cm
T_1 = 1 s	T_2 = not given = t (say)
n_1 = 4	n_2 = ?

Also $a = 1, b = -3, c = 0$

$$\text{As, } n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$
$$= 4\left[\frac{1\text{ g}}{100\text{ g}}\right]^1\left[\frac{1\text{ cm}}{10\text{ cm}}\right]^{-3}\left[\frac{1\text{s}}{t\text{ s}}\right]^0 = 4 \times \frac{1}{100} \times 1000$$
= 40 units

2. **(c) :** As $\frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{21}{20}$ ...(i)
If n is frequency of tuning fork, then
$n_1 - n = 5$ ...(ii)
$n - n_2 = 5$ ...(iii)
Adding eq. (ii) and (iii), we get $n_1 - n_2 = 10$
$n_1 = 10 + n_2$
$\frac{10 + n_2}{n_2} = \frac{21}{20}$ (From (i))
$\Rightarrow \frac{10}{n_2} + 1 = \frac{1}{20} + 1 \quad \therefore \quad n_2 = 200$
From (iii) $n = 5 + n_2 = 5 + 200 = 205$ Hz.

3. **(d):** Gravitational force of attraction on mass m at P due to solid sphere is

$$F=\frac{GMm}{(2R)^2}=\frac{GMm}{4R^2} \quad \text{or} \quad \frac{GMm}{R^2}=4F \qquad \text{...(i)}$$

Mass of the spherical portion removed from sphere

$$M'=\frac{M}{\frac{4}{3}\pi R^3}\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3=\frac{M}{8}$$

Gravitational force of attraction on mass m at P if mass of the spherical portion removed is present there is

$$F'=\frac{G(M/8)m}{\left(\frac{3R}{2}\right)^2}=\frac{GMm}{R^2}\times\frac{1}{8}\times\frac{4}{9}$$

$$F' = 4F\times\frac{1}{8}\times\frac{4}{9}=\frac{2F}{9} \qquad \text{(Using (i))}$$

$\therefore$ Gravitational force of attraction on mass m at P due to remaining part of the sphere is

$$F''=F-F'=F-\frac{2F}{9}=\frac{7F}{9}$$

4. **(d):** Mass of disc X, $m_X=\pi R^2 t\rho$

where ρ = density of material of disc.

$$\therefore \quad I_X=\frac{1}{2}m_X R^2=\frac{1}{2}\pi R^2 t\rho R^2 \quad \text{or} \quad I_X=\frac{1}{2}\pi\rho t R^4$$

Mass of disc Y, $m_Y=\pi(4R)^2\frac{t}{4}\rho=4\pi R^2 t\rho$

$$\therefore \quad I_Y=\frac{1}{2}m_Y(4R)^2=\frac{1}{2}4\pi R^2 t\rho(16R^2)$$

or $\quad I_Y=32\pi t\rho R^4$

$$\therefore \quad \frac{I_Y}{I_X}=\frac{32\pi t\rho R^4}{\frac{1}{2}\pi\rho t R^4}=64 \quad \text{or} \quad I_Y=64\,I_X$$

5. **(c):** The velocity acquired by the moving body is given by $v=u+at=0+3\times 30=90\text{ m s}^{-1}$

$\therefore$ Horizontal velocity of the projectile

$=v\cos\theta=90\text{ m s}^{-1}$

and the maximum height of the projectile is

$$h=\frac{v^2\sin^2\theta}{2g}$$

$$\therefore \quad v^2\sin^2\theta=2gh=2\times 10\times 80$$

$$\therefore \quad v\sin\theta=\sqrt{1600}=40\text{ m s}^{-1}$$

Thus for the projectile, $\dfrac{v\sin\theta}{v\cos\theta}=\dfrac{40}{90}=\dfrac{4}{9}$

$$\therefore \quad \tan\theta=\frac{4}{9} \quad \text{or} \quad \theta=\tan^{-1}\left(\frac{4}{9}\right)$$

6. **(d):** When the ball falls through a vertical height h, it acquires the velocity $v=\sqrt{2gh}$ $[v^2=u^2+2gh]$ and since this velocity does not change, when the ball travels through water, it must be its terminal velocity or critical velocity $(v_c)=\sqrt{2gh}$ ...(i)

$$\text{Also} \quad v_c=\frac{2}{9}\frac{r^2(\rho-\rho_{\text{water}})g}{\eta} \qquad \text{...(ii)}$$

From eqn. (i) and (ii), we get

$$\therefore \quad \sqrt{2gh}=\frac{2r^2}{9}\frac{(\rho-\rho_{\text{water}})g}{\eta}$$

$$\therefore \quad h=\left[\frac{2r^2}{9}\frac{(\rho-\rho_{\text{water}})g}{\eta}\right]^2\times\frac{1}{2g}$$

$$=\left[\frac{2\times(10^{-4})^2\times(10^4-10^3)\times 10}{9\times 10^{-5}}\right]^2\times\frac{1}{20}$$

$$\therefore \quad h=\frac{20\times 20}{20}=20\text{ m}$$

7. **(c):** Here, $V=100\text{ mL}=\dfrac{100\times 10^{-3}\times 1000}{(100)^3}\text{ m}^3$

$$=10^{-4}\text{ m}^3$$

$E=6.0\times 10^{-21}$ J, $P=200\times 10^3$ Pa $=2\times 10^5$ Pa

$$P=\frac{1}{3}\frac{mn}{V}C^2 \quad \text{or} \quad n=\frac{3PV}{mC^2}=\frac{3PV}{2E}$$

$$\left[\because E=\frac{1}{2}mC^2 \quad \text{or} \quad mC^2=2E\right]$$

$$\therefore \quad n=\frac{3\times(2\times 10^5)\times(10^{-4})}{2\times 6.0\times 10^{-21}}=5\times 10^{21}$$

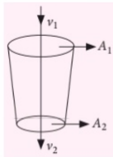
Number of moles $= \frac{n}{N_A} = \frac{5\times10^{21}}{6\times10^{23}} = 8.3\times10^{-3}$

8. (d): From the equation of continuity

$A_1v_1 = A_2v_2 \quad \therefore \quad v_2 = \frac{A_1v_1}{A_2}$ ...(i)

For the free falling flow of water,

$v_2^2 = v_1^2 + 2gh\,; \therefore\ v_2 = \sqrt{v_1^2 + 2gh}$ ...(ii)



Using eq. (ii) in (i), we get

$\sqrt{v_1^2 + 2gh} = \frac{A_1v_1}{A_2}$;

$\therefore \quad A_2 = \frac{A_1v_1}{\sqrt{v_1^2+2gh}}$

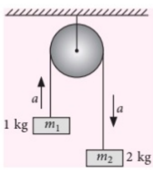
$A_2 = \frac{10^{-4}\times1}{\sqrt{1^2+2\times10\times0.15}} = 5\times10^{-5}\ \text{m}^2$

9. (a): $\because \quad m_2 > m_1$

The acceleration of the system,

$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{2-1}{1+2}\right)\times10 = \frac{10}{3}$

and the acceleration of the centre of mass



$a_{\text{cm}} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$

but $a_1 = a_2 = a$

$= \frac{1\times(-a)+2\times a}{1+2} = \frac{a}{3}$

$= \frac{10}{3\times3} = \frac{10}{9}$

$\therefore$ The distance travelled by the centre of mass in 2 second is (s) $= \frac{1}{2}a_{cm}t^2 = \frac{1}{2}\times\frac{10}{9}\times4$

$\therefore \quad s = \frac{20}{9}$ m.

10. (b): Let P and V be the initial pressure and volume of the gas.

For isothermal compression, $P_1 = P$, $V_1 = V$, $P_2 = 2P$, $V_2 = ?$

As $P_1V_1 = P_2V_2 \quad \therefore \quad PV = 2PV_2$

or $V_2 = \frac{PV}{2P} = \frac{V}{2}$

For adiabatic expansion, $P_1 = 2P$, $P_2 = 0.75P$, $V_1 = \frac{V}{2}, V_2 = V$

As $P_1V_1^\gamma = P_2V_2^\gamma \quad \therefore \quad (2P)\left(\frac{V}{2}\right)^\gamma = (0.75P)(V)^\gamma$

or $0.375 = \left(\frac{1}{2}\right)^\gamma$

On solving, we get $\gamma = 1.41$

11. (c): When particle is moving with uniform acceleration, let v be the velocity of particle at a distance s, then average velocity $= \frac{0+v}{2} = v/2$

time taken, $t_1 = \frac{s}{(v/2)} = \frac{2s}{v}$

When particle moves with uniform velocity,

time taken, $t_2 = \frac{2s}{v}$

When particle moves with uniform retardation,

time taken, $t_3 = \frac{3s}{(0+v)/2} = \frac{6s}{v}$

Total time $= t_1 + t_2 + t_3 = \frac{2s}{v} + \frac{2s}{v} + \frac{6s}{v} = \frac{10s}{v}$

$\therefore \quad v_{av} = \frac{s+2s+3s}{10s/v} \quad \Rightarrow \frac{v_{av}}{v} = \frac{6}{10} = \frac{3}{5}$

12. (b): As the bar is supported symmetrically by the three wires, each wire has the same increase in length (ΔL). Thus each wire has the same initial length (L) and same tension (T)

$\therefore \quad Y = \frac{TL}{A(\Delta L)}$

$Y \propto \frac{1}{A}$ or $Y \propto \frac{1}{\pi r^2}$ or $Y \propto \frac{1}{\pi\frac{D^2}{4}}$

$\therefore \quad D^2 \propto \frac{1}{Y}$ i.e., $D \propto \frac{1}{\sqrt{Y}}$

$\therefore \quad \frac{D_{\text{copper}}}{D_{\text{iron}}} = \sqrt{\frac{Y_{\text{iron}}}{Y_{\text{copper}}}} = \sqrt{\frac{1.9\times10^{11}}{1.1\times10^{11}}} = 1.3$

13. (d): For 16 g of helium, $n_1 = \frac{16}{4} = 4$

For 16 g of oxygen, $n_2 = \frac{16}{32} = \frac{1}{2}$

For mixture of gases,

$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$ where $C_V = \frac{f}{2}R$

$C_P = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$ where $C_P = \left(\frac{f}{2}+1\right)R$

For helium, $f_1 = 3$, $n_1 = 4$

For oxygen, $f_2 = 5$, $n_2 = \frac{1}{2}$

$$\therefore \quad \frac{C_P}{C_V} = \frac{\left(4\times\frac{5}{2}R\right)+\left(\frac{1}{2}\times\frac{7}{2}R\right)}{\left(4\times\frac{3}{2}R\right)+\left(\frac{1}{2}\times\frac{5}{2}R\right)} = \frac{47}{29} = 1.62$$

14. (b): Total mechanical energy, $E_T = 2$ J

For maximum speed, kinetic energy is maximum. The potential energy should therefore be minimum.

As $V(x) = \frac{x^4}{4} - \frac{x^2}{2}$

$$\therefore \quad \frac{dV}{dx} = \frac{4x^3}{4} - \frac{2x}{2} = x^3 - x = x(x^2-1)$$

For V to be minimum, $\frac{dV}{dx} = 0$

$\therefore \quad x(x^2 - 1) = 0$, or $x = 0, \pm 1$

At $x = 0$, $V(x) = 0$ and at $x = \pm 1$, $V(x) = -\frac{1}{4}$ J

$\therefore \quad (\text{Kinetic energy})_{max} = E_T - V_{min}$

or $\quad (\text{Kinetic energy})_{max} = 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$ J

or $\quad \frac{1}{2}mv_{max}^2 = \frac{9}{4}$ or $v_{max}^2 = \frac{9\times 2}{m\times 4} = \frac{9\times 2}{1\times 4} = \frac{9}{2}$

$\therefore \quad v_{max} = \frac{3}{\sqrt{2}}$ m s^{-1}

15. (a): As no external torque is applied to the system, the angular momentum of the system remains conserved.

$\therefore \quad L_i = L_f$

where the subscripts represent initial and final

or $\quad I_i\omega_i = I_f\omega_f$

Substituting the given values, we get

$\therefore \quad 1 \times 100 = 2 \times 1 \times \omega_f$ or $\omega_f = 50$ rad s^{-1} ...(i)

Initial kinetic energy, $K_i = \frac{1}{2}I_i\omega_i^2$

$= \frac{1}{2} \times 1 \times (100)^2 = 5 \times 10^3$ J

Final kinetic energy,

$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2} \times 2 \times 1 \times (50)^2$

$= 2.5 \times 10^3$ J (Using (i))

Loss in kinetic energy,

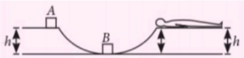
$\Delta K = K_i - K_f = 5 \times 10^3 - 2.5 \times 10^3$ J

$= 2.5 \times 10^3$ J $= 2.5$ kJ

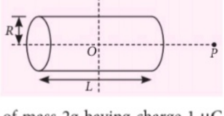
JEE WORK OUTS

1. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 m s^{-1} and the man behind walks at a speed 2.0 m s^{-1}. A third man is standing at a height 12 m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 m s^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. Find the frequency of beats heard by the stationary man at this instant.

2. The track shown in figure is frictionless. The block B of mass 2 m is lying at rest and the block A of mass m is pushed along the track with some speed. The collision between A and B is perfectly elastic. With what velocity should the block A be started to get the sleeping man awakened?



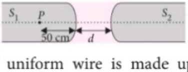
3. A non-conducting solid cylindrical rod of length L and radius R has uniformly distributed charge Q. Find the electric field at point P, a distance L from the centre of the rod.



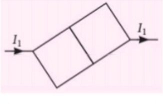
4. A ball of mass 2g having charge 1 μC suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Find the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution.

5. A pendulum clock is mounted in an elevator which starts going up at constant acceleration a, where $a < g$. At a height h the acceleration of the elevator reverses in direction, its magnitude remains constant. How soon after the start will the clock show the right time again?

6. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. Calculate the distance d.

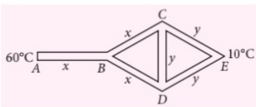


7. A piece of uniform wire is made up into two squares with a common side of length 4 inch. A current enters the squares at one of the corners and leaves at the diagonally opposite corner. What length of wire connected between input and output terminals would have an equivalent resistive effect?



8. A motorcycle and a car start from rest from the same place at the same time and travel in the same direction. The motorcycle accelerates at 1.0 m s^{-2} up to a speed of 36 km h^{-1} and the car at 0.5 m s^{-2} up to a speed of 54 km h^{-1}. Calculate the time and distance at whlch the car would overtake the motorcycle.

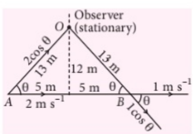
9. Three rods of material *x* and three of material *y* are connected as shown in figure. All the rods are identical in length and cross sectional area. If the end *A* is maintained at 60°C and the junction *E* at 10°C, calculate the temperature of the junction *B*. The thermal conductivity of *x* is 800 W m^{-1}°C^{-1} and of *y* is 400 W m^{-1}°C^{-1}.



10. A series AC circuit contains an inductor (20 mH), a capacitor (100 μF), a resistor (50 Ω) and an AC source of 12 V, 50 Hz. Find the energy dissipated in the circuit in 1000 s.

SOLUTIONS

1. Apparent frequency at *O* due to source at *A*,



$$\upsilon_A = \upsilon\left(\frac{v}{v-2\cos\theta}\right)$$

$$\upsilon_A = 1430\left[\frac{330}{330-2\cos\theta}\right]$$

$$\upsilon_A = 1430\left[\frac{1}{1-\frac{2\cos\theta}{330}}\right]$$

$$\approx 1430\left[1+\frac{2\cos\theta}{330}\right]$$ (Using Binomial expansion)

Apparent frequency at *O* due to source at *B*,

$$\upsilon_B = \upsilon\left[\frac{v}{v+1\cos\theta}\right]$$

$$\upsilon_B = 1430\left[\frac{330}{330+\cos\theta}\right] \approx 1430\left[1-\frac{\cos\theta}{330}\right]$$

(Using bionomial expansion)

Beat frequency $(\upsilon) = \upsilon_A - \upsilon_B = 1430\left[\frac{3\cos\theta}{330}\right]$

$$\upsilon = 13\cos\theta$$

From figure, $\cos\theta = \frac{5}{13}$ $\therefore$ $\upsilon = 13\left(\frac{5}{13}\right) = 5.00$ Hz

2. Let the velocity of $A = u_1$

The block *A* reaches to *B* before collision with velocity $= v_1$

$$\therefore \left(\frac{1}{2}\right)mv_1^2 - \left(\frac{1}{2}\right)mu_1^2 = mgh$$

$$\Rightarrow v_1^2 - u_1^2 = 2gh \Rightarrow v_1 = \sqrt{2gh+u_1^2} \quad ...(i)$$

When the block *B* reached at the upper man's head, the velocity of *B* is just zero.

For block *B*,

$$\therefore \left(\frac{1}{2}\right)\times 2m\times(0)^2 - \left(\frac{1}{2}\right)\times 2m\times v^2 = mgh$$

$$\Rightarrow v = \sqrt{2gh}$$

$\therefore$ Before collision velocity of $u_A = v_1$

$u_B = 0$

After collision velocity of $v_A = V$ (say)

$v_B = \sqrt{2gh}$

Since, it is an elastic collision the momentum and kinetic energy *K* be conserved.

$$\therefore m\times v_1 + 2m\times 0 = m\times v + 2m\times\sqrt{2gh}$$

$$\Rightarrow v_1 - v = 2\sqrt{2gh} \quad ...(i)$$

$$\text{Also } \left(\frac{1}{2}\right)\times m\times v_1^2 + \left(\frac{1}{2}\right)\times 2m\times(0)^2$$

$$= \left(\frac{1}{2}\right)\times mv^2 + \left(\frac{1}{2}\right)\times 2m(\sqrt{2gh})^2$$

$$\Rightarrow v_1^2 - v^2 = \sqrt{2gh}+\sqrt{2gh} \quad ...(ii)$$

Dividing (i) and (ii),

$$\frac{(v_1+v)(v_1-v)}{(v_1-v)} = \frac{2\times\sqrt{2gh}\times\sqrt{2gh}}{2\times\sqrt{2gh}}$$

$$\Rightarrow v_1 + v = \sqrt{2gh} \quad ...(iii)$$

Adding (i) and (iii),

$$2v_1 = 3\sqrt{2gh} \Rightarrow v_1 = \left(\frac{3}{2}\right)\sqrt{2gh}$$

$$\text{But } v_1 = \sqrt{2gh+u_1} = \left(\frac{3}{2}\right)\sqrt{2gh}$$

$$\Rightarrow 2gh + u_1^2 = \left(\frac{9}{4}\right)(2gh) \Rightarrow u_1 = \sqrt{2.5gh}$$

Thus the block A have to travel with a velocity greater than $\sqrt{2.5gh}$, so that man is awake.

3. Consider a disc of radius *R* of thickness *dr* at a distance *r* from the centre *O* of the cylinder.

Charge on the disc,

$$dq = \frac{Q}{\pi R^2 L}\times\pi R^2 dr = \frac{Q}{L}dr$$

$\because$ Electric field due to disc along its axis

$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(x^2+R^2)^{1/2}}\right]$$

$$\text{Hence, } dE = \frac{(Q/L)dr}{\pi R^2(2\varepsilon_0)}\left[1 - \frac{(L-r)}{[(L-r)^2+R^2]^{1/2}}\right]$$

$$E = \int dE = \int_{-L/2}^{+L/2} \frac{(Q/L)dr}{\pi R^2(2\varepsilon_0)}\left[1 - \frac{(L-r)}{[(L-r)^2+R^2]^{1/2}}\right]$$

$$E = \frac{Q}{2\pi R^2 L\varepsilon_0}\left[\int_{-L/2}^{+L/2} dr - \int_{-L/2}^{+L/2} \frac{(L-r)dr}{[(L-r)^2+R^2]^{1/2}}\right]$$

The second integral can be evaluated by substituting $(L - r)^2 + R^2 = t$.

Differentiating both sides, we get,

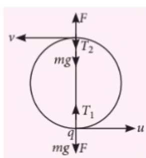
$$-2(L - r)dr = dt$$

$$\therefore \quad E = \frac{Q}{2\pi R^2 L\varepsilon_0}\left[[r]_{-L/2}^{+L/2} + \frac{1}{2}\int \frac{dt}{t^{1/2}}\right] = \frac{Q}{2\pi R^2 L\varepsilon_0}[L + \sqrt{t}]$$

$$E = \frac{Q}{2\pi R^2 L\varepsilon_0}\left[L + \left[\left[(L-r)^2+R^2\right]^{1/2}\right]_{-L/2}^{+L/2}\right]$$

$$E = \frac{Q}{2\pi R^2 L\varepsilon_0}\left[L + \left(\frac{L^2}{4}+R^2\right)^{1/2} - \left(\frac{9L^2}{4}+R^2\right)^{1/2}\right]$$

4. If the ball has to just complete the circle then the tension must vanish at the topmost point *i.e.*, $T_2 = 0$.



From Newton's second law,

$$T_2 + mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad \text{...(i)}$$

At the topmost point, $T_2 = 0$

$$\therefore \quad mg - \frac{q^2}{4\pi\varepsilon_0 l^2} = \frac{mv^2}{l} \quad \text{...(ii)}$$

From principle of energy conservation,

Energy at the lowest point = Energy at highest point

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg2l \quad \text{...(iii)}$$

From eqn. (ii),

$$v^2 = gl - \frac{q^2}{4\pi\varepsilon_0 ml} \quad \text{...(iv)}$$

From equation (iii) and (iv), using ($m = 2 \times 10^{-3}$ kg) we get,

$$u = \sqrt{5gl - \frac{q^2}{4\pi\varepsilon_0 ml}} = \sqrt{\frac{275}{8}} = 5.86 \text{ m s}^{-1} \simeq 6 \text{ m s}^{-1}$$

5. The frequency of the pendulum clock, when the elevator is at rest, is given by $\upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$.

The frequency when the elevator is accelerated up

$$\upsilon_1 = \frac{1}{2\pi}\sqrt{\frac{g+a}{l}}$$

$$\text{Time gain in one oscillation} = 2\pi\sqrt{\frac{l}{g}} - 2\pi\sqrt{\frac{l}{g+a}}$$

Time taken by elevator to travel a distance h up,

$$t = \sqrt{\frac{2h}{a}}$$

Number of oscillations in time $t = t \times \upsilon_1$

$$= \sqrt{\frac{2h}{a}} \times \frac{1}{2\pi}\sqrt{\frac{g+a}{l}}$$

$$\text{Time gained} = \sqrt{\frac{2h}{a}} \times \frac{1}{2\pi}\sqrt{\frac{g+a}{l}} \times \left[2\pi\sqrt{\frac{l}{g}} - 2\pi\sqrt{\frac{l}{g+a}}\right]$$

$$= \sqrt{\frac{2h}{a}}\left[\sqrt{\frac{g+a}{g}} - 1\right] = \sqrt{\frac{2h}{a}}[\sqrt{1+n} - 1] \quad \left[\text{Where, } n = \frac{a}{g}\right]$$

Let t_2 be the time in which the pendulum will lost the time it gained during up motion, so that it can keep correct time.

$$\text{Time lost in one oscillation} = 2\pi\sqrt{\frac{l}{g-a}} - 2\pi\sqrt{\frac{l}{g}}$$

$$\text{Frequency of oscillation} = \frac{1}{2\pi}\sqrt{\frac{g-a}{l}}$$

$$\text{Number of oscillation in time } t_2 = t_2 \times \frac{1}{2\pi}\sqrt{\frac{g-a}{l}}$$

$$\text{Time lost} = t_2 \times \frac{1}{2\pi}\sqrt{\frac{g-a}{l}}\left[2\pi\sqrt{\frac{l}{g-a}} - 2\pi\sqrt{\frac{l}{g}}\right]$$

$$= t_2\left[1 - \sqrt{\frac{g-a}{g}}\right] = t_2[1 - \sqrt{1-n}]$$

Time gained = Time lost

$$\sqrt{\frac{2h}{a}}[\sqrt{1+n} - 1] = t_2[1 - \sqrt{1-n}]$$

$$t_2 = \sqrt{\frac{2h}{a}}\left[\frac{\sqrt{1+n} - 1}{1 - \sqrt{1-n}}\right]$$

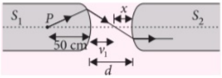
$$\text{Total time} = \sqrt{\frac{2h}{a}} + t_2 = \sqrt{\frac{2h}{a}}\left[\frac{\sqrt{1+n} - \sqrt{1-n}}{1 - \sqrt{1-n}}\right]$$

$$= \sqrt{\frac{2h}{a}}\left[\frac{\sqrt{1+\frac{a}{g}} - \sqrt{1-\frac{a}{g}}}{1 - \sqrt{1-\frac{a}{g}}}\right]$$

6. For glass rod S_1,

$R = -10$ cm, $\mu_1 = 1.5$, $\mu_2 = 1$, $u = -50$ cm, $v_1 = ?$

Ist refraction at curved surface of S_1 from denser medium to rarer medium, using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$$\frac{1}{v_1} - \frac{1.5}{-50} = \frac{1-1.5}{-10} \Rightarrow \frac{1}{v_1} = \frac{1}{20} - \frac{3}{100} = \frac{1}{50}$$

$\therefore v_1 = 50$ cm

For glass rod S_2, $R = 10$ cm, $u = -x$, $v = \infty$, $\mu_1 = 1$, $\mu_2 = 1.5$

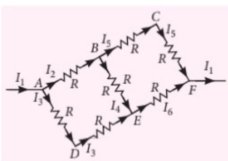
IInd refraction at curved surface of S_2 from rarer medium to denser,

$$\frac{1.5}{\infty} - \frac{1}{-x} = \frac{1.5-1}{10} = \frac{1}{20};\ \frac{1}{x} = \frac{1}{20} \quad \therefore\ x = 20 \text{ cm}$$

Separation between glass rods is given by

$d = v_1 + x = 50 + 20 = 70$ cm

7.



Let each side of double square have resistance R. Applying the Kirchhoff's 1st law, *i.e.* $\Sigma I = 0$

$$I_1 - I_2 - I_3 = 0 \qquad \text{...(i)}$$
$$I_2 - I_4 - I_5 = 0 \qquad \text{...(ii)}$$
$$I_3 + I_4 - I_6 = 0 \qquad \text{...(iii)}$$

Applying the Kirchhoff's 2nd rule, *i.e.* voltage rule to the loops *ABEDA*, and *BCFEB* gives

$$I_2R + I_4R - I_3 \times 2R = 0 \qquad \text{...(iv)}$$
$$I_5 \times 2R - I_6 \times R - I_4 \times R = 0 \qquad \text{...(v)}$$

Eliminating I_5 and I_6 from equations (ii), (iii) and (v), we obtain,

$$I_1 - I_2 - I_3 = 0 \qquad \text{...(vi)}$$
$$I_2 - 2I_3 + I_4 = 0 \qquad \text{...(vii)}$$
$$2I_2 - I_3 - 4I_4 = 0 \qquad \text{...(viii)}$$

Eliminating I_2 from these three equations, we get

$$I_1 - 3I_3 + I_4 = 0 \qquad \text{...(ix)}$$

and $2I_1 - 3I_3 - 4I_4 = 0$ ...(x)

or $I_4 = \frac{1}{5}I_1$ ...(xi)

The potential drop from *A* to *F* by *ADEFA* is

$$V_{AF} = I_3 \times 2R + I_6 \times R$$

Using equation (iii), we get

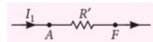
$$V_{AF} = R(2I_3 + I_3 + I_4)$$

Using equation (ix), we get

$V_{AF} = R(I_1 + 2I_4)$ and using (xi),

$$V_{AF} = I_1R\left(1 + \frac{2}{5}\right) = \frac{7}{5}RI_1$$

The equivalent effect is therefore obtained if a wire $\frac{7}{5}$ times the length of any side of the square is connected between *A* and *F*, because it produces the same potential drop as the double square between these points as shown in figure.



8. When car overtakes motorcycle, both have travelled the same distance in the same time. Let the total distance travelled be S and the total time taken to overtake be t.

For motor cycle :

Maximum speed attained= 36 km h^{-1}

$= \frac{36 \times 1000}{60 \times 60} = 10$ m s^{-1}

Since its acceleration = 1.0 m s^{-2}, the time t_1 taken by it to attain the maximum speed is given by

$v = u + at_1$ $(\because\ u = 0)$

$10 = 0 + 1.0 \times t_1$

$t_1 = 10$ s

The distance covered by motorcycle in attaining the maximum speed is

$$S_1 = 0 + \frac{1}{2}at_1 = \frac{1}{2} \times 1.0 \times (10)^2 = 50 \text{ m}$$

The time during which the motorcycle moves with maximum speed is $(t - 10)$ s.

The distance covered by the car during this time is

$S_1' = 10 \times (t - 10) = (10t - 100)$ m

Total distance travelled by motorcycle in time t is

$S = S_1 + S_1' = 50 + (10t - 100) = (10t - 50)$m ...(i)

For car :

Maximum speed attained = 54 km h^{-1}

$= \frac{54 \times 1000}{60 \times 60} = 15$ m s^{-1}

Since its acceleration = 0.5 m s^{-2}

The time taken by it to attain the maximum speed is given by

$15 = 0 + 0.5 \times t_2$ $(\because\ u = 0)$

or $t_2 = 30$ s

The distance covered by the car in attaining the maximum speed is

$$S_2 = 0 + \frac{1}{2}at_2^2 = \frac{1}{2} \times 0.5 \times (30)^2 = 225 \text{ m}$$

The time during which the car moves with maximum speed is $(t - 30)$ s.

The distance covered by the car during this time

$S_2' = 15 \times (t - 30) = (15t - 450)$ m

$\therefore$ Total distance travelled by car in time t is

$S = S_2 + S_2' = 225 + (15t - 450)$

$= (15t - 225)$ m ...(ii)

From equations (i) and (ii), we get

$10t - 50 = 15t - 225$ or $5t = 175$ or $t = 35$ s

On substituting this value of t in (i), we get

$S = 300$ m

9. It is clear from the symmetry of the figure that the points C and D are equivalent in all respect and hence, they are at the same temperature, say θ. No heat will flow through the rod CD. We can, therefore, neglect this rod in further analysis.

Let l and a be the length and the area of cross section of each rod. The thermal resistances of AB, BC and BD are equal. Each has a value

$$R_1 = \frac{1}{K_x}\frac{l}{a} \quad \text{...(i)}$$

Similarly, thermal resistances of CE and DE are equal, each having a value

$$R_2 = \frac{1}{K_y}\frac{l}{a} \quad \text{...(ii)}$$

As the rod CD has no effect, we can say that the rods BC and CE are joined in series. Their equivalent thermal resistance is

$R_3 = R_{BC} + R_{CE} = R_1 + R_2$.

Also, the rods BD and DE together have an equivalent thermal resistance $R_4 = R_{BD} + R_{DE} = R_1 + R_2$.

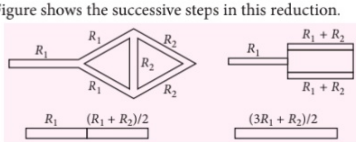
The resistances R_3 and R_4 are joined in parallel and hence their equivalent thermal resistance is given by

$$\frac{1}{R_5} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{2}{R_3} \quad \text{or,} \quad R_5 = \frac{R_3}{2} = \frac{R_1 + R_2}{2}$$

This resistance R_5 is connected in series with AB. Thus, the total arrangement is equivalent to a thermal resistance

$$R = R_{AB} + R_5 = R_1 + \frac{R_1 + R_2}{2} = \frac{3R_1 + R_2}{2}$$

Figure shows the successive steps in this reduction.



The heat current through A is

$$i = \frac{\theta_A - \theta_E}{R} = \frac{2(\theta_A - \theta_E)}{3R_1 + R_2}$$

This current passes through the rod AB. We have

$$i = \frac{\theta_A - \theta_E}{R_{AB}}$$

or $\theta_A - \theta_B = (R_{AB})i = R_1 \dfrac{2(\theta_A - \theta_E)}{3R_1 + R_2}$

Putting from (i) and (ii),

$$\theta_A - \theta_B = \frac{2K_y(\theta_A - \theta_E)}{K_x + 3K_y}$$

$$= \frac{2 \times 400}{800 + 3 \times 400} \times 50°\text{C} = 20°\text{C}$$

or $\theta_B = \theta_A - 20°\text{C} = 40°\text{C}$.

10. The time period of the source is

$$T = 1/\upsilon \text{ (frequency)} = \frac{1}{50} \text{ ms}$$

The given time 1000 s is much larger than the time period. Hence we can write the average power dissipated as

$P_{av} = V_{rms}\, i_{rms} \cos\phi$

where $\cos\phi = R/Z$ is the power factor. Thus,

$$P_{av} = V_{rms}\frac{V_{rms}}{Z}\frac{R}{Z} = \frac{RV_{rms}^2}{Z^2}$$

$$= \frac{(50\,\Omega)(12\text{ V})^2}{Z^2} = \frac{7200}{Z^2}\Omega\text{V}^2 \quad \text{...(i)}$$

The capcitance reactance $\dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}}\Omega$

$= \dfrac{100}{\pi}\Omega$

The inductive reactance $= \omega L = 2\pi \times 50 \times 20 \times 10^{-3}\ \Omega$

$= 2\pi\ \Omega$

The net reactance is $X = \dfrac{1}{\omega C} - \omega L = \dfrac{100}{\pi}\Omega - 2\pi\ \Omega$

$\approx 25.5\ \Omega$

Thus, $Z^2 = (50\ \Omega)^2 + (25.5\ \Omega)^2 = 3150\ \Omega^2$

From (i), average power $P_{av} = \dfrac{7200\ \Omega\text{V}^2}{3150\ \Omega^2} = 2.286$ W.

The energy dissipated in 1000 s $= P_{av} \times 1000$ s

$\approx 2.3 \times 10^3$ J

Monthly Test Drive CLASS XII ANSWER KEY

1. (c)	2. (a)	3. (d)	4. (a)	5. (c)
6. (a)	7. (c)	8. (d)	9. (d)	10. (b)
11. (b)	12. (d)	13. (b)	14. (a)	15. (c)
16. (a)	17. (b)	18. (b)	19. (a)	20. (a,c)
21. (a,c)	22. (a,b)	23. (a,b,c)	24. (1)	25. (9)
26. (3)	27. (c)	28. (b)	29. (b)	30. (a)

PRACTICE PAPER

NEET

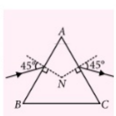
Exam on
26^{th} July 2020

1. In a watch glass, water is taken and it is kept in a non-uniform magnetic field. Water moves from strong field to weak field. The magnetic effect observed is
   (a) antiferromagnetic (b) ferromagnetic
   (c) diamagnetic (d) paramagnetic

2. If force (F), work (W) and velocity (v) are taken as the fundamental quantities, then the dimensions of time are
   (a) $[WFv]$ (b) $[WFv^{-1}]$
   (c) $[W^{-1}F^{-1}v]$ (d) $[WF^{-1}v^{-1}]$

3. When a body is projected vertically up from the ground, its potential energy and kinetic energy at a point P are in the ratio 2 : 3. If the same body is projected with double the previous velocity, then at the same point P, the ratio of its potential energy and kinetic energy would be
   (a) 9 : 1 (b) 1 : 9
   (c) 1 : 1 (d) 4 : 9

4. In the figure, what is the refractive index of material of the prism with respect to air, when the angle of deviation is 30°?



   (a) $\sqrt{3}$ (b) $\frac{3}{2}$
   (c) $\sqrt{2}$ (d) $\frac{4}{3}$

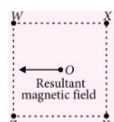
5. If $\vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = -5\hat{i} + 2\hat{j} - \hat{k}$, then what is the value of $(\vec{A} + \vec{B}) \cdot (\vec{A} \times 4\vec{B})$?
   (a) 96 (b) 48
   (c) 24 (d) 0

6. A pan pizza cools from 91°C to 79°C in 2 minutes, on a summer day, when the room temperature is 25°C. How long will the pan pizza take to cool from 91°C to 79°C, on a winter day, when the room temperature is 5°C?
   (a) $\frac{3}{2}$ minutes (b) 1 minute
   (c) $\frac{1}{2}$ minute (d) $\frac{1}{4}$ minute

7. A body of mass m has its position x at time t expressed by the equation $x = 3t^{3/2} + 2t + \frac{1}{2}$. The instantaneous force F on the body is proportional to
   (a) $t^{3/2}$ (b) t (c) t^0 (d) $t^{-1/2}$

8. Four parallel conductors, carrying equal currents, pass vertically through the four corners of a square $WXYZ$. In two conductors, the current is flowing into the page, and in the other two out of the page. In what directions must the currents flow to produce a resultant magnetic field in the direction shown at O, the centre of the square?

	Into the page	**Out of the page**
(a)	W and Y	X and Z
(b)	X and Z	W and Y
(c)	W and Z	X and Y
(d)	W and X	Y and Z

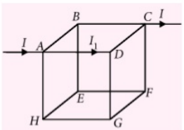


9. Number of nuclei of a radioactive substance are 1000 and 900 at times $t = 0$ and time $t = 2$ s. Then, number of nuclei at time $t = 4$ s will be
   (a) 800 (b) 810
   (c) 790 (d) 700

10. What is the path difference between the waves $y_1 = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and $y_2 = a\cos\left(\omega t - \frac{2\pi x}{\lambda}\right)$?

(a) λ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{4}$ (d) 2λ

11. 12 wires, each having resistance R are joined to form a skeleton cube as shown in the figure.



The current through wire BE is

(a) zero (b) $\frac{I}{3}$ (c) $I - I_1$ (d) $\frac{I_1}{2}$

12. Three point masses m_1, m_2, m_3 are located at the vertices of an equilateral triangle of length a. The moment of inertia of the system about an axis along the altitude of the triangle passing through m_1 is

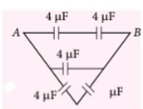
(a) $(m_2 + m_3)\frac{a^2}{4}$ (b) $(m_1 + m_2 + m_3)a^2$

(c) $(m_1 + m_2)\frac{a^2}{2}$ (d) $(m_2 + m_3)a^2$

13. Two particles P and Q describe simple harmonic motion of same amplitude A and frequency υ along the same straight line. The maximum distance between two particles is $\sqrt{3}A$. The initial phase difference between the particles is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

14. Equivalent capacitance between A and B in the figure is

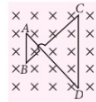


(a) 20 μF (b) 8 μF
(c) 12 μF (d) 16 μF

15. If the linear momentum of a body is increased by 50%, then the kinetic energy of that body increases by

(a) 100% (b) 125%
(c) 225% (d) 25%

16. A conducting wire frame is placed in a magnetic field which is directed into the plane of the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



(a) B to A and D to C
(b) A to B and C to D
(c) A to B and D to C
(d) B to A and C to D

17. A galvanometer has a coil of resistance 100 Ω and gives a full scale deflection for 30 mA current. If it is to work as a voltmeter of 30 V range, the resistance required to be added will be

(a) 900 Ω (b) 1800 Ω
(c) 500 Ω (d) 1000 Ω

18. The electric flux through a closed surface is zero. It means that

(a) There is no negative charges present inside the closed surface.
(b) there are no positive charges present inside the closed surface.
(c) the algebraic sum of all the charges present inside the surface is zero.
(d) the enclosed surface is a region of uniform electric field.

19. An object moves at a constant speed along a circular path in a horizontal XY plane, with the centre at the origin. When the object is at $x = -2$ m, its velocity is $-(4\text{ m s}^{-1})\hat{j}$. What is the object's acceleration when it is at $y = 2$ m?

(a) $-(8\text{ m s}^{-2})\hat{j}$ (b) $-(8\text{ m s}^{-2})\hat{i}$
(c) $-(4\text{ m s}^{-2})\hat{j}$ (d) $(4\text{ m s}^{-2})\hat{i}$

20. A large open tank has two holes in its wall. One is a square hole of side a at a depth of x from the top and the other is a circular hole of radius r at depth $4x$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then r is equal to

(a) $2\pi a$ (b) a

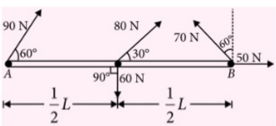
(c) $\frac{a}{2\pi}$ (d) $\frac{a}{\sqrt{2\pi}}$

21. A person covers one-third of the distance with 10 km h^{-1}, the second one-third distance with 20 km h^{-1} and the rest one-third distance with 60 km h^{-1}. What is the average speed?
(a) 12 km h^{-1} (b) 18 km h^{-1}
(c) 24 km h^{-1} (d) 30 km h^{-1}

22. When two spheres of equal masses undergo glancing elastic collision with one of them at rest, after collision they will move
(a) opposite to one another
(b) in the same direction
(c) together
(d) at right angle to each other

23. A source of frequency 500 Hz emits waves of wavelength 0.2 m. How long does it take the wave to travel 300 m?
(a) 70 s (b) 60 s (c) 12 s (d) 3 s

24. The total torque about pivot A provided by the forces shown in the figure, for $L = 3$ m, is



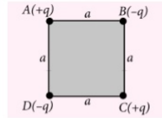
(a) 210 N m (b) 140 N m
(c) 95 N m (d) 75 N m

25. A motorboat covers the distance between the two spots on the river in 8 h and 12 h downstream and upstream respectively. The time taken by motorboat to cover this distance in still water is
(a) 7.6 h (b) 8.6 h (c) 9.6 h (d) 10.6 h

26. In a hydrogen atom, the electron is in n^{th} excited state. It comes down to the first excited state by emitting 10 different wavelengths. The value of n is
(a) 6 (b) 7 (c) 8 (d) 9

27. If the normal reaction is halved, the coefficient of friction will be
(a) unchanged (b) halved
(c) doubled (d) cannot be predicted

28. There are four point charges $+q$, $-q$, $+q$ and $-q$ placed at the corners A, B, C and D respectively of a square of side a. The potential energy of the system is $\frac{1}{4\pi\varepsilon_0}$ times



(a) $\frac{q^2}{a}(-4+\sqrt{2})$ (b) $\frac{q^2}{2a}(-4+\sqrt{2})$
(c) $\frac{4q^2}{a}$ (d) $\frac{-4\sqrt{2}q^2}{a}$

29. An ideal gas has an initial pressure of 3 pressure units and an initial volume of 4 volume units. The table gives the final pressure and volume of the gas (in those same units) in four processes. Which process starts and ends on the same isotherm?
(a) A
(b) B
(c) C
(d) D

	A	B	C	D
P	5	4	12	6
V	7	6	1	3

30. A planet moves round the sun in a circular orbit. The angular velocity of planet will be proportional to
(a) velocity of planet
(b) square of velocity of planet
(c) cube of velocity of planet
(d) none of these

31. The volume thermal expansion coefficient of an ideal gas at constant pressure is
(a) T (b) T^{-2} (c) T^{-1} (d) T^2

32. The frequency of a tuning fork A is 2% more than the frequency of a standard tuning fork. The frequency of a tuning fork B is 3% less than the frequency of the same standard tuning fork. When the tuning forks A and B are sounded together, 6 beats/second are heard. The frequency of the standard tuning fork is
(a) 120 Hz (b) 124 Hz
(c) 118 Hz (d) 130 Hz

33. Two identical metal plates show photoelectric effect. Light of wavelength λ_A falls on plate A and λ_B falls on plate B, $\lambda_A = 2\lambda_B$. The maximum kinetic energy of the photoelectrons are K_A and K_B respectively. Which one of the following is true?
(a) $2K_A = K_B$ (b) $K_A = 2K_B$
(c) $K_A < \frac{K_B}{2}$ (d) $K_A > 2K_B$

34. If emf $\varepsilon = 4\cos 1000t$ volt is applied to an L-R circuit of inductance 3 mH and resistance 4 Ω, the amplitude of current in the circuit is

(a) $\frac{4}{\sqrt{7}}$ A (b) 1.0 A

(c) $\frac{4}{7}$ A (d) 0.8 A

35. If x and y are the distances of an object and its image from the focus of a spherical mirror of focal length f, then $\frac{(xy)}{f^2}$ equal to

(a) 1 (b) 1.5 (c) 2 (d) 4

36. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If A is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is

(a) $\frac{W_1}{A}$ (b) $\frac{W_1 + (W/4)}{A}$

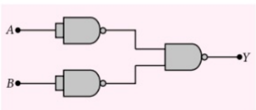
(c) $\frac{W_1 + (3W/4)}{A}$ (d) $\frac{W_1 + W}{A}$

37. A particle executes simple harmonic motion of type $x = A\sin\omega t$. It takes time t_1 from $x = 0$ to $x = \frac{A}{2}$ and t_2 from $x = \frac{A}{2}$ to $x = A$. The ratio $t_1 : t_2$ will be

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 2 : 1

38. The intensity of a plane electromagnetic wave is 5 W m^{-2}. It is incident normally on a perfectly reflecting surface. The radiation pressure is
(a) 3.33×10^{-6} N m^{-2}
(b) 3.33×10^{-7} N m^{-2}
(c) 3.33×10^{-8} N m^{-2}
(d) 3.33×10^{-9} N m^{-2}

39. Which logic gate is represented by the following combination of gates?



(a) OR (b) AND
(c) NAND (d) NOR

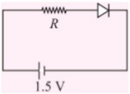
40. A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at $t = 18$ s. What is the value of v?
(a) 75 m s^{-1} (b) 55 m s^{-1}
(c) 40 m s^{-1} (d) 60 m s^{-1}

41. A stone thrown at an angle θ to the horizontal reaches a maximum height h. The time of flight of the stone is

(a) $\sqrt{\frac{2h\sin\theta}{g}}$ (b) $2\sqrt{\frac{2h\sin\theta}{g}}$

(c) $2\sqrt{\frac{2h}{g}}$ (d) $\sqrt{\frac{2h}{g}}$

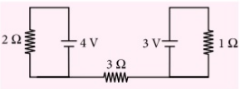
42. The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 milliwatt. What should be the value of the resistance R, connected in series with the diode for obtaining maximum current ?



(a) 1.5 Ω (b) 5 Ω
(c) 6.67 Ω (d) 200 Ω

43. If the distance between the first maxima and fifth minima of a double slit pattern is 7 mm and slits are separated by 0.15 mm with the screen 50 cm away from the slits, then the wavelength of light used is
(a) 600 nm (b) 525 nm
(c) 467 nm (d) 420 nm

44. The potential difference across the 3 Ω resistor shown in figure is



(a) zero (b) 1 V
(c) 3.5 V (d) 7 V

45. A 1 kg stone at the end of 1 m long string is whirled in a vertical circle at constant speed of 4 m s^{-1}. The tension in the string is 6 N, when the stone is at (Take $g = 10$ m s^{-2})
(a) top of the circle (b) bottom of the circle
(c) half way down (d) none of these

SOLUTIONS

1. **(c) :** When a diamagnetic substance is placed in a non-uniform magnetic field, it tends to move from strong field to weak field. As water moves from strong field to weak field, hence water is diamagnetic.

2. **(d):** Let $T \propto F^a W^b v^c$

or $\quad T = kF^a W^b v^c \quad$ ...(i)

where k is a dimensionless constant and a, b and c are exponents.

Writing the dimensions of various quantities on both sides, we get

$[M^0L^0T^1] = [MLT^{-2}]^a[ML^2T^{-2}]^b[LT^{-1}]^c$

$= [M^{a+b}L^{a+2b+c}T^{-2a-2b-c}]$

Applying the principle of homogeneity of dimensions, we get

$a + b = 0 \quad$ ...(ii)

$a + 2b + c = 0 \quad$ ...(iii)

$-2a - 2b - c = 1 \quad$ ...(iv)

On solving eqns. (i), (ii) and (iii), we get

$a = -1, b = 1$ and $c = -1$

From eqn. (i),

$T = kF^{-1}W^1v^{-1}$

Thus the dimensions of time are $[WF^{-1}v^{-1}]$.

3. **(b):** Let h be the height of point P above the ground. In both cases, potential energy of body at $P = mgh$.

In the first case,

As the ratio of potential energy and kinetic energy at P is 2 : 3,

$\therefore$ Kinetic energy at P

$= \frac{3}{2}$ potential energy at P

$= \frac{3}{2}mgh$

and total energy at $P = mgh + \frac{3}{2}mgh = \frac{5}{2}mgh$

If the body is projected vertically upwards from the ground with velocity u, then its kinetic energy at the point of projection

$= \frac{1}{2}mu^2$

By law of conservation of energy

$\frac{1}{2}mu^2 = \frac{5}{2}mgh \quad$ ...(i)

In the second case,

When the same body is projected with double the previous velocity (*i.e.* $2u$), then its kinetic energy at the point of projection

$= \frac{1}{2}m(2u)^2 = 4\left(\frac{1}{2}mu^2\right)$

$= 4\left(\frac{5}{2}mgh\right) = 10mgh \quad$ (using (i))

From the law of conservation of energy,

Total energy at $P = 10mgh$

But total energy at P

= kinetic energy at P + potential energy at P

$\therefore$ Kinetic energy at P

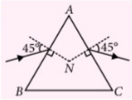
= total energy at P – potential energy at P

$= 10mgh - mgh = 9mgh$

Thus,

$$\frac{\text{Potential energy at } P}{\text{Kinetic energy at } P} = \frac{mgh}{9mgh} = \frac{1}{9}$$

4. **(c) :**



In the given figure,

Angle of incidence = angle of emergence

i.e., $i = e = 45°$

$\therefore$ This is the minimum deviation condition.

At the minimum deviation,

$i = \frac{A + \delta_m}{2}$ or $A = 2i - \delta_m$

where A is the angle of prism and δ_m is the angle of minimum deviation condition.

Here, $i = 45°, \delta_m = \delta = 30°$

$\therefore \quad A = 2(45°) - 30°$

$= 90° - 30° = 60°$

Let μ be the refractive index of material of the prism with respect to air. Then by prism formula

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60° + 30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

5. **(d):** Here, $\vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$,

$\vec{B} = -5\hat{i} + 2\hat{j} - \hat{k}$

$\therefore \quad \vec{A} + \vec{B} = (3\hat{i} - 2\hat{j} + 4\hat{k}) + (-5\hat{i} + 2\hat{j} - \hat{k}) = (-2\hat{i} + 3\hat{k})$

and $\vec{A} \times 4\vec{B} = 4(\vec{A} \times \vec{B})$

$$= 4\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ -5 & 2 & -1 \end{vmatrix}$$

$$= 4[\hat{i}(2-8) + \hat{j}(-20+3) + \hat{k}(6-10)]$$

$$= -24\hat{i} - 68\hat{j} - 16\hat{k}$$

$\therefore \quad (\vec{A}+\vec{B})\cdot(\vec{A}\times 4\vec{B})$

$$= (-2\hat{i} + 3\hat{k})\cdot(-24\hat{i} - 68\hat{j} - 16\hat{k})$$

$$= 48 + 0 - 48 = 0$$

6. (a): According to Newton's law of cooling

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$$

where T_s is the surrounding temperature.

For the first case,

$T_1 = 91°C, T_2 = 79°C, T_s = 25°C, t = 2$ min

$$\therefore \quad \frac{91°C - 79°C}{2 \text{ min}} = K\left(\frac{91°C + 79°C}{2} - 25°C\right)$$

or $\quad \dfrac{12°C}{2 \text{ min}} = K(60°C) \quad$ ...(i)

For the second case,

$T_1 = 91°C, T_2 = 79°C, T_s = 5°C, t = ?$

$$\therefore \quad \frac{91°C - 79°C}{t} = K\left(\frac{91°C + 79°C}{2} - 5°C\right)$$

or $\quad \dfrac{12°C}{t} = K(80°C) \quad$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{t}{2 \text{ min}} = \frac{60°C}{80°C} \text{ or } t = \frac{3}{4}(2 \text{ min}) = \frac{3}{2} \text{ min}$$

7. (d): As $x = 3t^{3/2} + 2t + \dfrac{1}{2}$

$$\therefore \quad \text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}\left(3t^{3/2} + 2t + \frac{1}{2}\right)$$

$$= 3\left(\frac{3}{2}\right)t^{1/2} + 2 + 0 = \frac{9}{2}t^{1/2} + 2$$

and acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{9}{2}t^{1/2} + 2\right) = \frac{9}{2}\left(\frac{1}{2}\right)t^{-1/2} + 0 = \frac{9}{4}t^{-1/2}$$

By Newton's second law

$$F = ma = m\left(\frac{9}{4}t^{-1/2}\right) \text{ or } F \propto t^{-1/2}$$

8. (d): To get resultant magnetic field in shown direction, current in W should be in and that in Y should be out; current in X should be in and that in Z should be out.

9. (b): In 2 s only 90% nuclei are left behind. Thus, in next 2 s 90% of 900 or 810 nuclei will be left.

10. (c): Given :

$$y_1 = a\sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \text{ and}$$

$$y_2 = a\cos\left(\omega t - \frac{2\pi x}{\lambda}\right) = a\sin\left(\frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda}\right)$$

The phase of the first wave is

$$\phi_1 = \omega t - \frac{2\pi x}{\lambda}$$

and that of the second wave is

$$\phi_2 = \frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda}$$

The phase difference between the waves is

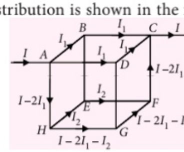
$$\Delta\phi = \phi_2 - \phi_1$$

$$= \left[\frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda}\right] - \left[\omega t - \frac{2\pi x}{\lambda}\right] = \frac{\pi}{2}$$

The corresponding path difference between the waves is

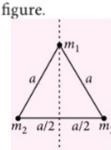
$$\Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{\lambda}{2\pi}\left(\frac{\pi}{2}\right) = \frac{\lambda}{4}$$

11. (a): According to Kirchhoff's junction law, the current distribution is shown in the figure.



From the figure due to symmetrys it is clear that no current flows in BE. Hence the current through wire BE is zero.

12. (a): The situation is as shown in figure.



Perpendicular distance of m_2 and m_3 from altitude of triangle = $a\cos 60° = \dfrac{a}{2}$

$\therefore$ Moment of inertia of the system about the given axis is

$$I = m_1(0)^2 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2 = (m_2 + m_3)\frac{a^2}{4}$$

13. (a): Let ϕ be the initial phase difference between the particles. Then

$x_1 = A\sin\omega t$ and $x_2 = A\sin(\omega t + \phi)$

The distance between them is

$$\Delta x = x_2 - x_1 = A\sin(\omega t + \phi) - A\sin\omega t$$

$$= 2A\cos\left(\frac{\omega t + \phi + \omega t}{2}\right)\sin\left(\frac{\omega t + \phi - \omega t}{2}\right)$$

$$\left(\because\ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right)$$

$$= 2A\cos\left(\omega t + \frac{\phi}{2}\right)\sin\frac{\phi}{2}$$

For the maximum value, $\cos\left(\omega t + \frac{\phi}{2}\right) = 1$

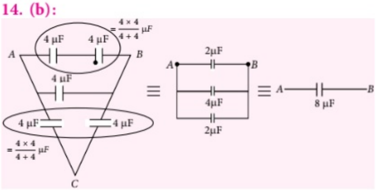
$\therefore\quad \Delta x_{max} = 2A\sin\frac{\phi}{2}$

But $\Delta x_{max} = \sqrt{3}A$ (given)

$\therefore\quad \sqrt{3}A = 2A\sin\frac{\phi}{2}$

or $\sin\frac{\phi}{2} = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$ or $\frac{\phi}{2} = \frac{\pi}{3}$ or $\phi = \frac{2\pi}{3}$

14. (b):



$C_{AB} = 8\ \mu F$

15. (b): The kinetic energy of the body is $K = \frac{p^2}{2m}$

where p is the linear momentum and m is the mass of the body.

Since mass remains constant, so

$$K \propto p^2 \qquad \therefore\ \frac{K'}{K} = \left(\frac{p'}{p}\right)^2$$

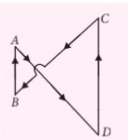
But $p' = p + \frac{50}{100}p = \frac{3}{2}p$

$$\therefore\ \frac{K'}{K} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

% increase in the kinetic energy $= \frac{K' - K}{K} \times 100\%$

$$= \left(\frac{K'}{K} - 1\right)\times 100\% = \left(\frac{9}{4} - 1\right)\times 100\% = 125\%$$

16. (a): Magnetic field in ⊗ direction is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus, current in both the loops should be anticlockwise.



But as the area of the loop on the right side is more, induced emf in this side will be more compared to the left side loop.

Therefore, net current in the complete loop will be in a direction shown in figure.

17. (a): Here, Resistance of the galvanometer, $G = 100\ \Omega$

Current for full scale deflection, $I_g = 30$ mA $= 30 \times 10^{-3}$ A

In order to work this galvanometer as a voltmeter of range 30 V (*i.e.* V = 30 V), let resistance R be added in series with it. It is given by

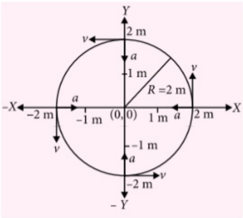
$$R = \frac{V}{I_g} - G = \frac{30\ \text{V}}{30\times10^{-3}\ \text{A}} - 100\ \Omega$$

$$= 1000\ \Omega - 100\ \Omega = 900\ \Omega$$

18. (c)

19. (a): Here, Radius of circular path, $R = 2$ m

Speed of the object, $v = 4\ \text{m s}^{-1}$



The magnitude of acceleration is

$$a = \frac{v^2}{R} = \frac{(4\ \text{m s}^{-1})^2}{2\ \text{m}} = 8\ \text{m s}^{-2}$$

The acceleration is directed towards the centre.

Therefore, when the object is at y = 2 m, its acceleration is $-8\hat{j}\ \text{m s}^{-2}$.

20. (d): Speed of the water coming out from the hole is $v = \sqrt{2gh}$

where h denotes depth of the hole from the free surface of the water in the tank.

The quantities of water flowing out per second from both holes are given to be same, therefore

$$A_1v_1 = A_2v_2$$

where A_1 and A_2 are the areas of cross-section of hole 1 (*i.e.* square hole) and hole 2 (*i.e.* circular hole) respectively and v_1 and v_2 are the speeds of water coming out from these holes.

$$\therefore \quad a^2\sqrt{2gx} = \pi r^2\sqrt{2g4x}$$

$$\text{or} \quad a^2 = 2\pi r^2 \quad \text{or} \quad r^2 = \frac{a^2}{2\pi} \quad \text{or} \quad r = \frac{a}{\sqrt{2\pi}}$$

21. (b): Let s be the total distance covered by the person.

If t_1, t_2 and t_3 are times taken by the person to cover first one-third distance with 10 km h^{-1}, second one-third with 20 km h^{-1} and the rest one-third with 60 km h^{-1} respectively, then

$$t_1 = \frac{s/3}{10} = \frac{s}{30},\ t_2 = \frac{s/3}{20} = \frac{s}{60} \text{ and}$$

$$t_3 = \frac{s/3}{60} = \frac{s}{180}$$

The average speed is

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{s}{t_1+t_2+t_3} = \frac{s}{\frac{s}{30}+\frac{s}{60}+\frac{s}{180}}$$

$$= \frac{s}{\frac{s}{30}\left(1+\frac{1}{2}+\frac{1}{6}\right)} = \frac{1}{\frac{1}{30}\left(\frac{6+3+1}{6}\right)}$$

$$= \frac{180}{10} = 18 \text{ km h}^{-1}$$

22. (d)

23. (d): Here, Frequency, $\upsilon = 500$ Hz

Wavelength, $\lambda = 0.2$ m

The velocity of the wave is

$$v = \upsilon\lambda = (500 \text{ Hz})(0.2 \text{ m}) = 100 \text{ m s}^{-1}$$

Time taken by the wave to travel 300 m is

$$t = \frac{300 \text{ m}}{100 \text{ m s}^{-1}} = 3 \text{ s}$$

24. (d): Resolve the 90 N, 80 N and 70 N forces into x and y components. The line of action of 90 N, 50 N and x components of the 80 N and 70 N forces pass through the pivot point A, therefore they cause no rotation.

$\therefore$ The total torque about point A is

$$\tau_A = (80\sin 30°)\left(\frac{L}{2}\right) - 60\left(\frac{L}{2}\right) + (70\cos 60°)(L)$$

But $L = 3$ m (given)

$$\therefore \quad \tau_A = (80)\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) - (60)\left(\frac{3}{2}\right) + (70)\left(\frac{1}{2}\right)(3)$$

$$= (60 - 90 + 105) \text{ N m} = 75 \text{ N m}$$

25. (c): Let v_b and v_w be the velocity of motorboat in still water and velocity of water flow in the river respectively. If x is the distance between the two spots, then

$$v_b + v_w = \frac{x}{8} \text{ (for downstream)} \quad \text{...(i)}$$

$$v_b - v_w = \frac{x}{12} \text{ (for upstream)} \quad \text{...(ii)}$$

On adding eqns. (i) and (ii), we get

$$2v_b = \frac{x}{8} + \frac{x}{12} = \frac{20}{96}x$$

$$\text{or} \quad v_b = \frac{10x}{96}$$

Time taken by motorboat to cover the distance x in still water is

$$t = \frac{x}{v_b} = \frac{x}{\frac{10x}{96}} = \frac{96}{10} = 9.6 \text{ h}$$

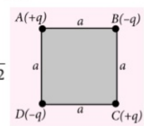
26. (a): Number of possible emission lines are $n(n-1)/2$ when an electron jumps from n^{th} state to ground state. In this question, this value should be $(n-1)(n-2)/2$.

$$\text{Hence, } 10 = \frac{(n-1)(n-2)}{2}$$

On solving this, we get $n = 6$

27. (a): The coefficient of friction depends only on the nature of the surfaces in contact.

28. (a): Given :



$$AB = BC = CD = AD = a$$

$$\therefore \quad AC = BD = \sqrt{a^2 + a^2} = a\sqrt{2}$$

The total potential energy of the system is

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(q)(-q)}{AB} + \frac{(q)(q)}{AC} + \frac{(q)(-q)}{AD} + \frac{(-q)(q)}{BC} + \frac{(-q)(-q)}{BD} + \frac{(q)(-q)}{CD}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{a} + \frac{q^2}{a\sqrt{2}} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{a\sqrt{2}} - \frac{q^2}{a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{4q^2}{a} + \frac{2q^2}{a\sqrt{2}}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}[-4+\sqrt{2}]$$

29. (c): For the same isotherm,

$$P_iV_i = P_fV_f$$

where subscripts i and f refer to initial and final.

Among the given four processes, only C satisfy this condition.

30. (c) : The velocity of planet is

$$v = \sqrt{\frac{GM}{r}}$$

where M is the mass of the sun and r is the radius of the orbit.

Squaring both sides, we get

$$v^2 = \frac{GM}{r} \quad \text{or} \quad r = \frac{GM}{v^2} \qquad \text{...(i)}$$

The angular velocity of planet is

$$\omega = \frac{v}{r} = \frac{v}{GM/v^2} = \frac{v^3}{GM} \qquad \text{(using (i))}$$

Thus, $\omega \propto v^3$

31. (c) : According to an ideal gas equation

$$PV = nRT \qquad \text{...(i)}$$

At constant pressure

$$PdV = nRdT \qquad \text{...(ii)}$$

Dividing eqn. (ii) by eqn. (i), we get

$$\frac{dV}{V} = \frac{dT}{T}$$

or $$dV = \frac{V}{T}dT \qquad \text{...(iii)}$$

If γ represents volume thermal expansion coefficient of an ideal gas, then

$$dV = \gamma V dT \qquad \text{...(iv)}$$

Equating eqns. (iii) and (iv), we get

$$\gamma V dT = \frac{V}{T}dT \quad \text{or} \quad \gamma = \frac{1}{T} = T^{-1}$$

32. (a) : Let the frequency of standard fork be υ. Then as per question

$$\upsilon_A = \upsilon + \frac{2}{100}\upsilon = \frac{102}{100}\upsilon$$

and $$\upsilon_B = \upsilon - \frac{3}{100}\upsilon = \frac{97}{100}\upsilon$$

When tuning forks A and B are sounded together, 6 beats are heard per second.

$\therefore \quad \upsilon_A - \upsilon_B = 6$

or $$\frac{102}{100}\upsilon - \frac{97}{100}\upsilon = 6 \text{ or } \upsilon = \frac{6\times 100}{5} = 120 \text{ Hz}$$

33. (c) : According to Einstein's photoelectric equation,

$$K_A = \frac{hc}{\lambda_A} - \phi_0 \text{ and } K_B = \frac{hc}{\lambda_B} - \phi_0$$

But $\lambda_A = 2\lambda_B$ (given)

$$\therefore \quad K_A = \frac{hc}{2\lambda_B} - \phi_0 = \frac{1}{2}[K_B + \phi_0] - \phi_0$$

or $$K_A = \frac{K_B}{2} - \frac{\phi_0}{2} \quad \therefore \quad K_A < \frac{K_B}{2}$$

34. (d): Here, $L = 3$ mH $= 3 \times 10^{-3}$ H, $R = 4\ \Omega$

On comparing $\varepsilon = 4\cos 1000t$ with $\varepsilon = \varepsilon_0 \cos\omega t$, we get

$\varepsilon_0 = 4$ V and $\omega = 1000$ rad s^{-1}

The amplitude of current in the circuit is

$$I_0 = \frac{\varepsilon_0}{Z} = \frac{\varepsilon_0}{\sqrt{R^2 + X_L^2}} = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L)^2}}$$

$$= \frac{4}{\sqrt{4^2 + (1000\times 3\times 10^{-3})^2}} = \frac{4}{5} = 0.8 \text{ A}$$

35. (a): According to mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Here, $u = f + x$, $v = f + y$, $f = f$

$$\therefore \quad \frac{1}{f+x} + \frac{1}{f+y} = \frac{1}{f}$$

or $$\frac{f+y+f+x}{(f+x)(f+y)} = \frac{1}{f}$$

or $f^2 + fy + f^2 + fx = f^2 + fy + fx + xy$

or $2f^2 = f^2 + xy$ or $f^2 = xy$

Thus, $\dfrac{xy}{f^2} = 1$

36. (c) : Total force at height $3L/4$ from its lower end

= Weight suspended + Weight of 3/4 of the wire

$= W_1 + (3W/4)$

Hence, stress $= \dfrac{W_1 + (3W/4)}{A}$

37. (b): Here, $x = A\sin\omega t$

At $t = t_1$, $x = \dfrac{A}{2}$

$$\therefore \quad \frac{A}{2} = A\sin\omega t_1 \quad \text{or} \quad \sin\omega t_1 = \frac{1}{2}$$

or $$\omega t_1 = \frac{\pi}{6} \quad \text{or} \quad t_1 = \frac{\pi}{6\omega} \qquad \text{...(i)}$$

At $t = t_1 + t_2$, $x = A \qquad \therefore A = A\sin\omega(t_1 + t_2)$

or $\sin\omega(t_1 + t_2) = 1$

or $$\omega(t_1 + t_2) = \frac{\pi}{2} \quad \text{or} \quad t_1 + t_2 = \frac{\pi}{2\omega}$$

or $\quad t_2 = \frac{\pi}{2\omega} - t_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{2\pi}{6\omega} \qquad$ (using (i))

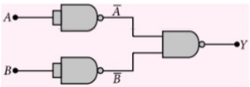
Thus, $\frac{t_1}{t_2} = \frac{1}{2}$

38. (c) : The radiation pressure on a perfectly reflecting surface is

$$P = \frac{2I}{c}$$

Here, $I = 5\ \text{W m}^{-2}$, $c = 3\times10^8\ \text{m s}^{-1}$

$$\therefore \quad P = \frac{2(5\ \text{W m}^{-2})}{3\times10^8\ \text{m s}^{-1}} = 3.33\times10^{-8}\ \text{N m}^{-2}$$

39. (a) : 

The Boolean expression for output Y is

$Y = \overline{\overline{A}\cdot\overline{B}}$ (by de Morgan's theorem)

$= \overline{\overline{A}} + \overline{\overline{B}} = A + B$

which is Boolean expression for OR gate. Thus the combination represents OR gate.

40. (a) : For first ball,

$u = 0, a = g = 10\ \text{m s}^{-2}, t = 18\ \text{s}$

As $\quad s = ut + \frac{1}{2}at^2$

$$\therefore \quad s_1 = 0 + \frac{1}{2}(10\ \text{m s}^{-2})(18\ \text{s})^2 = 1620\ \text{m}$$

For second ball,

$u = v, a = g = 10\ \text{m s}^{-2}, t = 18\ \text{s} - 6\ \text{s} = 12\ \text{s}$

$$\therefore \quad s_2 = ut + \frac{1}{2}at^2 = v(12\ \text{s}) + \frac{1}{2}(10\ \text{m s}^{-2})(12\ \text{s})^2$$

$= v(12\ \text{s}) + 720\ \text{m}$

At the time of meeting, $s_2 = s_1$

$\therefore \quad v(12\ \text{s}) + 720\ \text{m} = 1620\ \text{m}$

or $\quad v = \frac{1620\ \text{m} - 720\ \text{m}}{12\ \text{s}} = 75\ \text{m s}^{-1}$

41. (c) : Let the stone be thrown with velocity u at an angle θ to the horizontal. Then

Maximum height $h = \frac{u^2\sin^2\theta}{2g}$

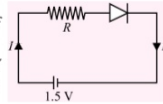
or $\quad \sqrt{2gh} = u\sin\theta \qquad$ ...(i)

The time of flight of the stone is

$$T = \frac{2u\sin\theta}{g} = \frac{2}{g}\sqrt{2gh} = 2\sqrt{\frac{2h}{g}} \qquad \text{(using (i))}$$

42. (b): Here, voltage drop across diode,

$V_D = 0.5$ V



Maximum power rating of diode,

$P = 100\ \text{mW} = 100\times10^{-3}\ \text{W}$

The current in the diode is

$$I = \frac{P}{V_D} = \frac{100\times10^{-3}\ \text{W}}{0.5\ \text{V}} = 0.2\ \text{A}$$

The voltage drop across R,

$V_R = 1.5\ \text{V} - 0.5\ \text{V} = 1\ \text{V}$

The resistance $R = \frac{V_R}{I} = \frac{1\ \text{V}}{0.2\ \text{A}} = 5\ \Omega$

43. (a) : Here,

Distance between the slits, $d = 0.15\ \text{mm} = 0.15\times10^{-3}\ \text{m}$

Distance of screen from the slits, $D = 50$ cm

$= 50\times10^{-2}$ m

Since there are three and a half fringes contained within 7 mm (*i.e.* from first maxima to fifth minima),

$\therefore \quad \frac{7}{2}\beta = 7$ mm (where β is the fringe width)

or $\quad \beta = \frac{14\ \text{mm}}{7} = 2\ \text{mm}$

As $\quad \beta = \frac{\lambda D}{d}$

$$\therefore \quad \lambda = \frac{\beta d}{D} = \frac{(2\times10^{-3}\ \text{m})(0.15\times10^{-3}\ \text{m})}{(50\times10^{-2}\ \text{m})}$$

$= 6\times10^{-7}\ \text{m} = 600\times10^{-9}\ \text{m}$

$= 600\ \text{nm} \quad (\because 1\ \text{nm} = 10^{-9}\ \text{m})$

44. (a)

45. (a) : Here, $m = 1$ kg, $r = 1$ m, $v = 4\ \text{m s}^{-1}$, $T = 6$ N

Let the string make an angle θ with the vertical. Then

$$T - mg\cos\theta = \frac{mv^2}{r}$$

or $\quad mg\cos\theta = T - \frac{mv^2}{r} = 6\ \text{N} - \frac{(1\ \text{kg})(4\ \text{m s}^{-1})^2}{1\ \text{m}}$

$= -10$ N

or $\quad \cos\theta = \frac{-10\ \text{N}}{mg} = \frac{-10\ \text{N}}{(1\ \text{kg})(10\ \text{m s}^{-2})} = -1$ or $\theta = 180°$

It means the stone is at the top of the vertical circle.

◈◈

Monthly Test Drive CLASS XI ANSWER KEY

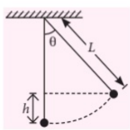
1. (b)	2. (a)	3. (a)	4. (a)	5. (b)
6. (b)	7. (a)	8. (c)	9. (d)	10. (b)
11. (a)	12. (a)	13. (b)	14. (d)	15. (a)
16. (d)	17. (a)	18. (a)	19. (b)	20. (b,c)
21. (a,b,c)	22. (a,c)	23. (b,c,d)	24. (3)	25. (9)
26. (2)	27. (b)	28. (a)	29. (d)	30. (c)

GET SET GO for JEE

with exclusive and brain storming MCQs

Practicing these MCQs help to strengthen your concepts and give you extra edge in your JEE preparation

1. A simple pendulum with bob of mass m and conducting wire of length L swings under gravity through an angle 2θ. The earth's magnetic field component in the direction perpendicular to swing is B.



Maximum potential difference induced across the pendulum is

(a) $2BL\sin\left(\frac{\theta}{2}\right)(gL)^{1/2}$ (b) $BL\sin\left(\frac{\theta}{2}\right)(gL)$

(c) $BL\sin\left(\frac{\theta}{2}\right)(gL)^{3/2}$ (d) $BL\sin\left(\frac{\theta}{2}\right)(gL)^2$

2. A beam of light, consisting of two wavelengths 560 nm and 420 nm, is used to obtain interference fringes in a Young's double slit experiment. Find the least distance from the central maxima, where the bright fringes due to both the wavelengths coincide. The distance between the two slits is 4 mm and the screen is at a distance of 1 m from the slits.

(a) 2.21 mm (b) 0.42 mm

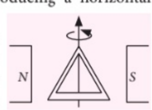
(c) 5.22 mm (d) 2 cm

3. When a metal surface is illuminated by light of wavelengths 400 nm and 250 nm, the maximum velocities of the photoelectrons ejected are v and $2v$ respectively. The work function of the metal is (h = Planck's constant, c = velocity of light in air)

(a) $2\,hc \times 10^6$ J (b) $1.5\,hc \times 10^6$ J

(c) $hc \times 10^6$ J (d) $0.5\,hc \times 10^6$ J

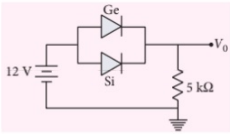
4. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 A is passed through it and the magnetic field is parallel to its plane.



(a) $5\sqrt{2}$ N m (b) $5\sqrt{2} \times 10^{-4}$ N m

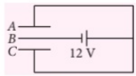
(c) $5\sqrt{3} \times 10^{-7}$ N m (d) 10^{-7} N m

5. Ge and Si diodes conduct at 0.3 V and 0.7 V, respectively. In the following figure, if Ge diode connection are reversed, the value of V_0 changes by



(a) 0.2 V (b) 0.4 V (c) 0.6 V (d) 0.8 V

6. Three plates A, B and C each of area 50 cm^2 have separation 3 mm between A and B and 6 mm between B and C.



The energy stored when the plates are fully charged by a 12 V battery is

(a) 2 μJ (b) 1.6 nJ (c) 5 μJ (d) 3.2 nJ

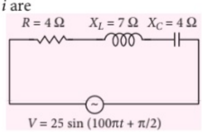
7. In an experiment with two coherent sources, the amplitude of the intensity variation is found to be 5% of the average intensity. What will be the ratio of intensities of the light waves of interfering source.

(a) 400 : 1 (b) 500 : 1 (c) 500 : 3 (d) 1600 : 1

8. A motor is being self started against a resistance torque of 60 N m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead acid battery. If the battery has a capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.

(a) 14 (b) 24 (c) 36 (d) 42

9. For the series LCR circuit as shown in figure, the heat developed in 80 s and amplitude of wattless current i are



(a) 4000 J, 3 A (b) 8000 J, 3 A
(c) 4000 J, 4 A (d) 8000 J, 5 A

10. The wavelength of the first line of Balmer series is 6563 Å. The wavelength of the first line of Lyman series is

(a) 1215 Å (b) 4861 Å (c) 4340 Å (d) 4101 Å

11. Electromagnetic waves travel in a medium with a speed of 2×10^8 m s^{-1}. The relative magnetic permeability of the medium is 1. Find the relative electrical permittivity.

(a) 2.25 (b) 4.01 (c) 0.52 (d) 12.2

12. Two circular coils of wires made of similar wires but of radius 20 cm and 40 cm are connected in parallel. The ratio of the magnetic fields at their centres is

(a) 4 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

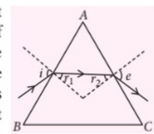
13. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{\tau}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$, it experiences a torque $\vec{\tau}_2 = -\vec{\tau}_1$. The angle θ is

(a) 30° (b) 45° (c) 60° (d) 90°

14. At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half-lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances?

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

15. In the given figure, for what value of $\angle i$ should a ray of light be incident on the face of a prism of refracting angle 60°, so that it just suffers total internal reflection at other face?



(For prism, μ = 1.524)

(a) 15.12° (b) 22.12° (c) 29.74° (d) 32.21°

SOLUTIONS

1. **(a) :** As $h = L(1 - \cos\theta)$

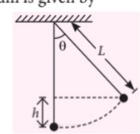
Maximum velocity at equilibrium is given by

$\therefore \; v^2 = 2gh$

$v^2 = 2gL(1 - \cos\theta)$

$v^2 = 2gL\left(2\sin^2\frac{\theta}{2}\right)$

$v = 2\sqrt{gL}\sin\left(\frac{\theta}{2}\right)$



Thus the maximum potential difference,

$$V_{max} = BvL = B \times 2\sqrt{gL}\sin\frac{\theta}{2}L = 2BL\sin\left(\frac{\theta}{2}\right)(gL)^{1/2}$$

2. **(b) :** To find the point of coincidence of bright fringes, we can equate the distance of bright fringes from the central maxima, made by both the wavelengths of light.

Given, distance between the screen and slit, $D = 1$ m

Slit width, $d = 4 \times 10^{-3}$ m

$\lambda_1 = 560$ nm, $\lambda_2 = 420$ nm

Let nth order bright fringe of λ_1 coincides with $(n + 1)$th order bright fringe of λ_2.

$\therefore \quad \frac{Dn\lambda_1}{d} = \frac{D(n+1)\lambda_2}{d} \; (\lambda_1 > \lambda_2)$

$\Rightarrow \; n\lambda_1 = (n+1)\lambda_2 \Rightarrow \frac{n+1}{n} = \frac{\lambda_1}{\lambda_2}$

$\therefore \quad 1 + \frac{1}{n} = \frac{560\times10^{-9}}{420\times10^{-9}} \Rightarrow 1 + \frac{1}{n} = \frac{4}{3}$

$\therefore \; n = 3$

$\therefore$ Least distance from the central fringe where bright fringe of two wavelengths coincides

= distance of 3rd order bright fringe of λ_1

$\therefore \; y_n = \frac{3D\lambda_1}{d} = \frac{3\times1\times560\times10^{-9}}{4\times10^{-3}} = 0.42\times10^{-3}$ m

$\therefore \; y_n = 0.42$ mm

3rd bright fringe of λ_1 and 4th bright fringe of λ_2 coincide at 0.42 mm from central fringe.

3. **(a) :** By using $\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2$

$$\Rightarrow \frac{hc}{400\times10^{-9}} = W_0 + \frac{1}{2}mv^2 \quad \ldots\text{(i)}$$

and $$\frac{hc}{250\times10^{-9}} = W_0 + \frac{1}{2}m(2v)^2 \quad \ldots\text{(ii)}$$

On solving eqns. (i) and (ii), we get

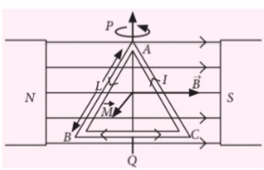
$$\frac{1}{2}mv^2 = \frac{hc}{3}\left[\frac{1}{250\times10^{-9}} - \frac{1}{400\times10^{-9}}\right] \quad \ldots\text{(iii)}$$

From eqns. (i) and (iii), we get $W_0 = 2\,hc\times10^6$ J.

4. **(c) :** As the coil is in the form of an equilateral triangle, its area,

$$A = \frac{1}{2}L\times L\sin 60° = \frac{1}{2}\times(0.02)^2\times\frac{\sqrt{3}}{2}$$

$$A = \sqrt{3}\times10^{-4}\text{ m}^2$$



And so its magnetic moment

$M = IA = 0.1\times\sqrt{3}\times10^{-4} = \sqrt{3}\times10^{-5}$ A m^2

Now the couple on a current carrying coil in a magnetic field is given by

$\vec{\tau} = \overline{M}\times\overline{B}$ *i.e.*, $\tau = MB\sin\theta$

and when the plane of the coil is parallel to the magnetic field, the angle between $\overline{M}$ (*i.e.*, normal to the area of coil) and $\overline{B}$ will be 90° and hence

$\tau = MB\sin 90° = MB$

i.e., $\tau = (\sqrt{3}\times10^{-5})\times5\times10^{-2} = 5\sqrt{3}\times10^{-7}$ N m

5. **(b) :** Equivalent voltage drop across the combination Ge and Si diode = 0.3 V

$$\Rightarrow \text{Current, } i = \frac{(12-0.3)\text{ V}}{5\text{ k}\Omega} = 2.34\text{ mA}$$

$\therefore$ Output voltage, $V_0 = Ri = 5\text{ k}\Omega\times2.34\text{ mA} = 11.7$ V

Now consider the case when Ge diode connection are reversed. In this case voltage drop across the diode's combination = 0.7 V

$$\Rightarrow \text{Current, } i = \frac{(12-0.7)\text{ V}}{5\text{ k}\Omega} = 2.26\text{ mA}$$

$\therefore \; V_0 = iR = 2.26\text{ mA}\times5\text{ k}\Omega = 11.3$ V

Hence change in the value of $V_0 = 11.7 - 11.3 = 0.4$ V

6. **(b) :**

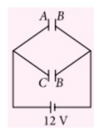
$$C_{AB} = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2})(50\times10^{-4}\,\text{m}^2)}{(3\times10^{-3}\,\text{m})}$$

$$= 14.75\times10^{-12}\text{ F}$$

$$C_{BC} = \frac{(8.85\times10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2})(50\times10^{-4}\,\text{m}^2)}{(6\times10^{-3}\,\text{m})}$$

$$= 7.375\times10^{-12}\text{ F}$$

$\therefore$ The equivalent capacitance of the arrangement is



$C_{eq} = C_{AB} + C_{BC}$

$= 14.75\times10^{-12}\text{ F} + 7.375\times10^{-12}\text{ F}$

$= 22.125\times10^{-12}$ F

The energy stored when the plates are fully charged by a 12 V battery is

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(22.125\times10^{-12}\text{ F})(12\text{ V})^2$$

$$= 1593\times10^{-12}\text{ J} \approx 1.6\times10^{-9}\text{ J} = 1.6\text{ nJ}$$

7. **(d)**: As we know that, $I = I_{av} \pm 0.05\, I_{av}$

$\Rightarrow I_{max} = I_{av}(1 + 0.05) = 1.05\, I_{av}$ ...(i)

$I_{min} = I_{av}(1 - 0.05) = 0.95\, I_{av}$ ...(ii)

From eqn. (i) and (ii), we get

$$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{1.05}{0.95} \Rightarrow \left(\frac{r+1}{r-1}\right)^2 = \frac{105}{95}$$

$\Rightarrow (r^2 + 1 + 2r)95 = 105(r^2 + 1 - 2r)$

$\Rightarrow 10r^2 - 400r + 10 = 0 \Rightarrow r = 40$

$\therefore \frac{I_1}{I_2} = r^2 = (40)^2 = 1600 : 1$

8. **(b)**: Angular speed, $\omega = \frac{2\pi \times 75}{60} = 2.5\,\pi \text{ rad s}^{-1}$

Power required for cranking the engine at a speed of 75 r.p.m

$P = T \times \omega = 60 \times 2.5\,\pi = 471$ W

Energy required per start = Power × Time per start

$= 471 \times 8 = 3{,}768$ watt seconds or joules $= \frac{3{,}768}{3{,}600}$

$= 1.047$ Wh

Energy drawn from the battery

$$= \frac{\text{Energy required per start}}{\text{Overall efficiency of motor and gear}}$$

$= \frac{1.047}{0.25} = 4.18$ Wh

Number of starts that can be made with 100 Wh capacity battery $= \frac{100}{4.18} = 24$ (approx.)

9. **(a)**: As heat produced in an LCR circuit, *i.e.*, $H = (i_{rms})^2 Rt$,

The rms value of current, $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{25}{Z\sqrt{2}}$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

On putting the given values of R, X_L and X_C, we get $Z = 5\,\Omega$

$$H = \left(\frac{25}{5\sqrt{2}}\right)^2 \times 4 \times 80 = 4000 \text{ J}$$

Amplitude of wattless current is $i_0 \sin\phi$

where $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \Rightarrow \phi \approx 37°$

Amplitude of wattless current

$= i_0 \sin 37° = \frac{25}{5} \times \frac{3}{5} = 3$ A

10. **(a)**: For first line of Balmer series,

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \text{ or } \frac{1}{\lambda} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

or $\frac{1}{\lambda} = R\left[\frac{9-4}{36}\right] = \frac{5R}{36}; \lambda = \frac{36}{5R}$ ...(i)

For first line of Lyman series, $\frac{1}{\lambda'} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$$\frac{1}{\lambda'} = R\left[1 - \frac{1}{4}\right]$$

$\frac{1}{\lambda'} = \frac{3R}{4}$ or $\lambda' = \frac{4}{3R}$ ...(ii)

From eqns. (i) and (ii), we get

$$\frac{\lambda'}{\lambda} = \frac{4}{3R} \times \frac{5R}{36} = \frac{20}{108}$$

$$\lambda' = \frac{20}{108} \times 6563 \text{ Å} = 1215 \text{ Å}$$

11. **(a)**: Given, $v = 2 \times 10^8$ m s^{-1} and $\mu_r = 1$

The speed of electromagnetic waves in a medium is given by $v = \frac{1}{\sqrt{\mu\varepsilon}}$ ...(i)

where, μ and ε are absolute permeability and absolute permittivity of the medium.

Now, $\mu = \mu_0\mu_r$ and $\varepsilon = \varepsilon_0\varepsilon_r$

$\therefore$ Eq. (i) becomes, $v = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$

$$= \frac{1}{\sqrt{\mu_0\varepsilon_0}} \times \frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

$v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$ $\left[\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$

On squaring both sides, we get

$$\varepsilon_r = \frac{c^2}{v^2\mu_r} = \frac{(3\times10^8)^2}{(2\times10^8)^2\times1} = 2.25$$

12. **(a)**: When coils are connected in parallel, then ratio of currents flowing in these coils will be equal to reciprocal of the ratio of their resistances.

$$\therefore \frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{\rho(l_2/A)}{\rho(l_1/A)}$$

(as both the coils are made of similar wires)

$\Rightarrow \frac{l_2}{l_1} = \frac{2\pi r_2}{2\pi r_1} = \frac{r_2}{r_1}$ $\therefore \frac{B_1}{B_2} = \frac{[(\mu_0 i_1)/2r_1]}{[(\mu_0 i_2)/2r_2]}$

$\frac{B_1}{B_2} = \frac{i_1}{i_2} \times \frac{r_2}{r_1} = \frac{r_2^2}{r_1}$ $\left[\because \frac{i_1}{i_2} = \frac{r_2}{r_1}\right]$

$$= \left(\frac{40}{20}\right)^2 = \frac{4}{1} = 4$$

13. **(c)**: Dipole moment of fixed dipole can be written as, $\vec{p} = p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}$

For electric field, $\vec{E}_1 = E\hat{i}$

Torque on the dipole

$$\vec{\tau}_1 = (\vec{p}\times\vec{E}_1) \Rightarrow \vec{\tau}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j})\times(E\,\hat{i})$$

$$\vec{\tau}_1 = pE\sin\theta\,(-\hat{k}) \quad \ldots\text{(i)}$$

Now, for $\vec{E}_2 = \sqrt{3}E_1\hat{j} = \sqrt{3}E\hat{j}$

In this case, torque on the dipole, $\vec{\tau}_2 = (\vec{p}\times\vec{E}_2)$

$$\vec{\tau}_2 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j})\times(\sqrt{3}E\hat{j})$$

$$\vec{\tau}_2 = \sqrt{3}pE\cos\theta\,(\hat{k}) \quad \ldots\text{(ii)}$$

Now given, $\vec{\tau}_2 = -\vec{\tau}_1$

$$\sqrt{3}pE\cos\theta\,(\hat{k}) = -pE\sin\theta\,(-\hat{k})$$

$$\sqrt{3}\cos\theta = \sin\theta \text{ or } \frac{\sin\theta}{\cos\theta} = \sqrt{3};\ \tan\theta = \sqrt{3}$$

$\Rightarrow \theta = 60°$

14. (a) : Number of half-lives in two days for substance 2 and 1 respectively are $n_1 = \frac{2\times24}{12} = 4$ and $n_2 = \frac{2\times24}{16} = 3$

By using $N = N_0\left(\frac{1}{2}\right)^n$

$$\Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2}\times\frac{\left(\frac{1}{2}\right)^{n_1}}{\left(\frac{1}{2}\right)^{n_2}} = \frac{2}{1}\times\frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^3} = \frac{1}{1}$$

15. (c) : For total internal reflection, angle of emergence, $e = 90°$. Now applying Snell's law at face AC, we have

$$\frac{\sin e}{\sin r_2} = \mu \Rightarrow \sin r_2 = \frac{1}{\mu}\times\sin 90°$$

$$\Rightarrow \sin r_2 = \frac{1}{1.524}\times 1$$

$$\Rightarrow r_2 = \sin^{-1}(0.656)$$

$$\Rightarrow r_2 = 41°$$

From geometry, $\angle A = \angle r_1 + \angle r_2$

$$\angle r_1 = \angle A - \angle r_2 = 60° - 41° = 19°$$

Applying Snell's law at face AB, we have

$$\mu = \frac{\sin i}{\sin r_1} \Rightarrow \sin i = (\mu)\sin r_1$$

$$\Rightarrow \sin i = 1.524\times\sin 19°$$

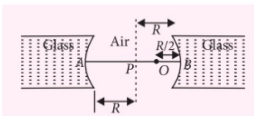
$$\Rightarrow i = \sin^{-1}(0.496) \Rightarrow i = 29.74°$$

GEAR UP FOR JEE MAIN

EXAM ON 18th-23rd July 2020

with Numerical Value Type Questions

1. Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object O is placed as shown in figure. What is the separation between the images of O formed by each refracting surface?

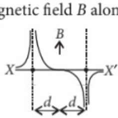


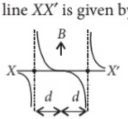
(a) $114R$ (b) $1.0114R$
(c) $0.114R$ (d) $0.4R$

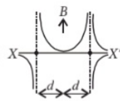
2. If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula
(a) $[P^1A^{-1}T^1]$ (b) $[P^2A^1T^1]$
(c) $[P^1A^{-1/2}T^1]$ (d) $[P^1A^{1/2}T^{-1}]$

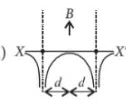
3. A system consists of a uniformly charged sphere of radius R and a surrounding medium filled by a charge with the volume density $\rho = \frac{\alpha}{r}$, where α is a positive constant and r is the distance from the centre of the sphere. Find the charge of the sphere for which the electric field intensity E outside the sphere is independent of R.
(a) $\frac{\alpha}{2\epsilon_0}$ (b) $\frac{2}{\alpha\epsilon_0}$
(c) $2\pi\alpha R^2$ (d) None of these

4. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the line XX' is given by

(a) 

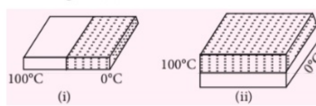
(b) 

(c) 

(d) 

5. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of microsecond another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?
(a) One photon of energy 10.2 eV and an electron of energy 1.4 eV.
(b) Two photons of energy 1.4 eV.
(c) Two photons of energy 10.2 eV.
(d) One photon of energy 10.2 eV and another photon of 1.4 eV.

6. Two identical rectangular rods of metal of thermal resistance R, are welded end to end as shown in figure (i) and 10 J of heat flows through the rods in 2 min. How long would it take for 30 J of heat to flow through the the rods if they are welded as shown in figure (ii).

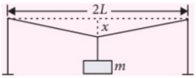


(a) 2 min (b) 1.5 min
(c) 1 min (d) 4 min

7. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. The minimum initial speed for the body to reach the surface of smaller star is
(a) $\frac{2}{3}\sqrt{\frac{Gm}{a}}$ (b) $\frac{3}{2}\sqrt{\frac{5Gm}{a}}$

(c) $\frac{2}{3}\sqrt{\frac{5Gm}{a}}$ (d) $\frac{3}{2}\sqrt{\frac{Gm}{a}}$

8. A mild-steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars,



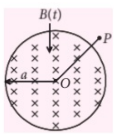
A mass m is suspended from the midpoint of the wire. Strain in the wire is

(a) $\frac{x^2}{2L^2}$ (b) $\frac{x}{L}$ (c) $\frac{x^2}{L}$ (d) $\frac{x^2}{2L}$

9. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(a) 0.42 m from mass of 0.3 kg
(b) 0.70 m from mass of 0.7 kg
(c) 0.98 m from mass of 0.3 kg
(d) 0.98 m from mass of 0.7 kg.

10. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown in the figure. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region

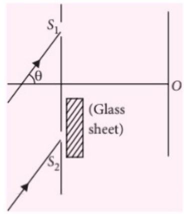


(a) is zero (b) decreases as $\frac{1}{r}$
(c) increases as r (d) decreases as $\frac{1}{r^2}$.

11. An insulated container containing n moles of monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

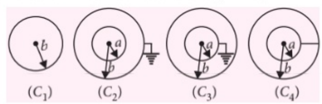
(a) $\frac{mv_0^2}{3R}$ (b) $\frac{mv_0^2}{3nR}$
(c) $\frac{mnv_0^2}{R}$ (d) $\frac{mv_0^2}{2R}$

12. A monochromatic beam of light falls on Young's double slit experiment apparatus at some angle (say θ) as shown in figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference = 0) will be obtained



(a) anywhere depending on angle θ, thickness of plate t and refractive index of glass μ
(b) at O
(c) below O (d) above O

13. Consider the following four arrangement of spherical shells of radius a and b ($a << b$).



Then which of the following holds good for the value of their capacitances.

(a) $C_3 < C_2 > C_1 = C_4$ (b) $C_3 > C_1 = C_4 > C_2$
(c) $C_2 > C_3 > C_1 > C_4$ (d) $C_2 > C_1 = C_4 > C_3$

14. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$
(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$.

15. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statements is true?

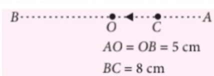
(a) Error ΔT in measuring T, the time period, is 0.02 seconds.
(b) Error ΔT in measuring T, the time period, is 1 second.
(c) Percentage error in the determination of g is 5%.
(d) Percentage error in the determination of g is 2.5%.

16. The ratio of contributions made by the electric field and magnetic field components to the intensity of an e.m. wave is
(a) $c:1$ (b) $c^2:1$ (c) $1:1$ (d) $\sqrt{c}:1$

17. The water flows from a tap of diameter 1.25 cm with a rate of 5×10^{-5} m^3 s^{-1}. The density and coefficient of viscosity are 10^3 kg m^3 and 10^{-3} Pa s respectively. The flow of water is
(a) steady with Reynolds number 5100
(b) turbulent with Reynolds number 5100
(c) steady with Reynolds number 3900
(d) turbulent with Reynolds number 3900

18. Two bulbs consume same power when operated at 200 V and 300 V respectively. When the two bulbs are connected in series across a DC source of 500 V, then ratio of
(a) potential difference across them is 3 : 2
(b) potential difference across them is 2 : 3
(c) power consumed by them is 4 : 9
(d) power consumed by them is 2 : 3

19. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart, as shown in figure. Take the direction from A to B as the +ve direction and choose the incorrect statement.

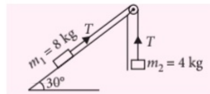


(a) The signs of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive.
(b) The signs of velocity of the particle at C going towards O is negative.
(c) The signs of velocity, acceleration and force on the particle when it is 4 cm away from B going towards A are negative.
(d) The signs of acceleration and force on the particle when it is at point B are negative.

20. The electron emitted in beta radiation originates from
(a) inner orbits of atoms
(b) free electrons existing in nuclei
(c) decay of a neutron in a nuclei
(d) photon escaping from the nucleus.

NUMERICAL VALUE TYPE

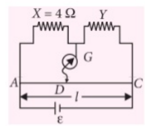
21. Two masses are connected by a string as shown in the figure over a frictionless pulley.



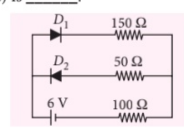
The acceleration (in m s^{-2}) of the system is ______.

22. The magnetic field of a beam emerging from a filter facing a floodlight is given by
$B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$T.
What is the average intensity (in W m^{-2}) of the beam?

23. Figure shows a meter bridge, wire AC has uniform cross-section. The length of wire AC is 100 cm. X is a standard resistor of 4 Ω and Y is a coil. When Y is immersed in melting ice, the null point is at 40 cm from point A. When the coil Y is heated to 100°C, a 100 Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point. Temperature coefficient (10^{-4} K^{-1}) of resistance of the coil is ______.



24. The circuit shown in the figure contains two diodes each with a forward resistance of 50 ohm and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 ohm resistance (in ampere) is ______.



25. In refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power, and heat is transferred from –3°C to 27°C, find the heat (in kJ) taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

SOLUTIONS

1. **(c) :** For image formed by the surface on the right side

$$\mu_1 = 1, \mu_2 = 1.5, u = \frac{-R}{2}, R_1 = -R$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

$$\frac{1.5}{v} + \frac{2}{R} = \frac{1.5-1}{-R} \Rightarrow v = \frac{-3R}{5}$$

The image is at a distance $\left(R - \frac{3R}{5}\right) = 0.4R$ from the centre P towards the right hand side. For the surface on the left hand side

$$u = \frac{-3R}{2}, R_2 = -R, \mu_1 = 1, \mu_2 = 1.5$$

So, $\frac{1.5}{v} + \frac{2}{3R} = \frac{1.5-1}{-R}$ or $v = \frac{-9R}{7}$

The image is at a distance of $\left(\frac{9R}{7} - R\right) = \frac{2R}{7}$ from the centre P towards the right hand side.

$\therefore$ The distance between the two images is

$$= 0.4R - \frac{2R}{7} = 0.114\,R$$

Therefore answer is (c).

2. **(d):** Let energy, $E \propto P^a A^b T^c$

or $E = kP^a A^b T^c$

or $[ML^2T^{-2}] = [MLT^{-1}]^a [L^2]^b [T]^c$
$= [M^a L^{a+2b} T^{-a+c}]$

whence, $a = 1, b = \frac{1}{2}, c = -1$

Dimensional formula for E is $[P^1 A^{1/2} T^{-1}]$.

3. **(c) :** Using Gauss theorem for spherical surface of radius r outside the sphere with the charge q

$$\int_0^r E ds = \frac{1}{\varepsilon_0}\int_0^r \rho(r)\,dV$$

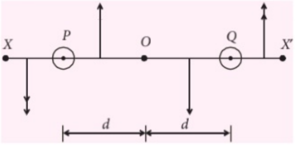
$$E4\pi r^2 = \frac{1}{\varepsilon_0}\left(q + \int_R^r \frac{\alpha}{r}(4\pi r^2)dr\right)$$

$$E4\pi r^2 = \frac{(q - 2\pi\alpha R^2)}{\varepsilon_0} + \frac{4\pi\alpha r^2}{2\varepsilon_0}$$

The intensity E does not depend on R if

$$\frac{q - 2\pi\alpha R^2}{\varepsilon_0} = 0 \text{ or } q = 2\pi\alpha R^2$$

4. **(b):** The steady currents in the two parallel wires flow out of paper. O is a point in the centre between two wires. The wires are at P and Q. Since the current flows out of the plane of the paper, the magnetic field at points to the right of given wire will be upwards and to the left will be downwards.



(i) At point O, net field will be zero. The fields due to wires at P and Q are equal in magnitude but opposite in directions.
(ii) Region QX' : Magnetic field will be upwards. The region lies to right of both the wires.
(iii) Region OQ : Net field will be downwards. Due to nearness, wire at Q dominates.
(iv) Region OP : Net field will be upwards. Due to nearness, wire at P dominates.
(v) Region PX : Net field will be downwards. The region lies to left of both the wires.

Graph (b) satisfies all the five conditions.

5. **(a) :** For a hydrogen atom,

$$E_1 = -13.6\text{ eV}, E_2 = \frac{13.6}{2^2} = -3.4\text{ eV}$$

$\therefore$ $E_2 - E_1 = -3.4 + 13.6 = 10.2$ eV

During inelastic collision, photon of energy 10.2 eV is absorbed by the hydrogen atom. This excites the electron from ground state to the second orbit. The electron jumps back to the ground state in less than a microsecond releasing a photon of energy 10.2 eV. The second photon of 15 eV energy will ionise the atom because ionisation energy is just 13.6 eV. The knocked out electron will retain the balance energy = 15 – 13.6 = 1.4 eV.

6. **(b):** The thermal resistance of each rod is R.
In case (i) the rods are in series.

$\therefore \quad R_S = R + R = 2R$

Rate of flow of heat will be

$$\frac{dQ}{dt} = \frac{\Delta T}{R_S};\ \frac{10}{2} = \frac{(100-0)}{2R}$$

$\therefore \quad R = 10$

In case (ii) the rods are parallel; so

$$\frac{1}{R_P} = \frac{1}{R} + \frac{1}{R} \Rightarrow R_P = \frac{R}{2} = 5$$

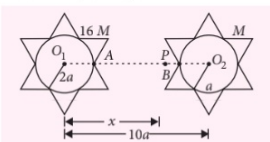
Now, rate of flow of heat in this case will be

$$\frac{dQ}{dt} = \frac{\Delta T}{R_P} \Rightarrow \frac{30}{t} = \left(\frac{100-0}{5}\right)$$

or $\quad t = 1.5$ minute.

7. (b) : First we have to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow \quad \frac{G(16M)}{x^2} = \frac{GM}{(10a-x)^2} \Rightarrow x = 8a(\text{from } O_1)$$

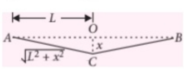


i.e., once the body reaches P, the gravitational pull of attraction due to M takes the lead to make m move towards it automatically as the gravitational pull of attraction due to $16M$ vanishes i.e., a minimum KE or velocity has to be imparted to m from surface of $16M$ such that it is just able to overcome the gravitational pull of $16M$. By law of conservation of energy

$$\frac{1}{2}mv^2 + \left[-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right] = 0 + \left[-\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right]$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - \frac{GMm}{8a}(45) \Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

8. (a) :

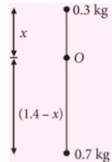


Change in length, $\Delta L = AC - AO$

$$= [L^2 + x^2]^{1/2} - L = L\left[1 + \frac{1}{2}\frac{x^2}{L^2}\right] - L = \frac{x^2}{2L}$$

Longitudinal strain $= \frac{\Delta L}{L} = \frac{x^2/2L}{L} = \frac{x^2}{2L^2}$

9. (c) : Let the axis of rotation pass through O.



$I = mr^2$ for point mass.

$\therefore \quad I = I_1 + I_2$

$= 0.3\,x^2 + 0.7\,(1.4 - x)^2$

$= 0.3x^2 + 0.7\,(1.96 + x^2 - 2.8x)$

$= x^2 + 1.372 - 1.96x$

The work done for rotation of the rod is stored as rotational kinetic energy, $\frac{1}{2}I\omega^2$, of rod.

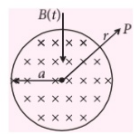
or $\quad W = \frac{I\omega^2}{2} = \frac{1}{2}(x^2 + 1.372 - 1.96x)\omega^2$

For work done to be minimum, $\frac{dW}{dx} = 0$

$$\therefore \quad \frac{d}{dx}\left[(x^2 + 1.372 - 1.96x)\right]\frac{\omega^2}{2} = 0$$

or $\quad 2x + 0 - 1.96 = 0$ or $2x = 1.96$ or $x = 0.98$ m

10. (b) : Magnetic field $B(t)$ is directed into the plane of the paper. P lies outside the field.



$$\oint \vec{E}\cdot d\vec{l} = \frac{d\phi}{dt}$$

or $\quad E(2\pi r) = \frac{d}{dt}(\vec{B}\cdot\vec{A})$

or $\quad 2\pi rE = \pi a^2\left(\frac{dB}{dt}\right)\cos 0°$

or $\quad E = \frac{a^2}{2r}\frac{dB}{dt}$ or $E \propto \frac{1}{r}$

11. (a) : There is n moles of the monoatomic gas in the container.

molar mass of the gas $= m$,

Total mass of the gas in the container, $M = mn$

Change in KE of the gas when the container is suddenly stopped, *i.e.*,

$$\Delta K = (\text{KE})_{\text{initial}} - (\text{KE})_{\text{final}} = \frac{1}{2}Mv_0^2 - 0$$

$$= \frac{1}{2}Mv_0^2 = \frac{1}{2}mnv_0^2$$

This change in kinetic energy (ΔK) result in a change in internal energy (ΔU) of the gas.

$$\Delta U = nC_V \Delta T = n\left(\frac{3}{2}R\right)\Delta T$$

Here, ΔT is the change in temperature of the gas.

As $\Delta U = \Delta K,\ n\left(\frac{3}{2}R\right)\Delta T = \frac{1}{2}mnv_0^2$

$\therefore\ \Delta T = \frac{mv_0^2}{3R}$

12. (a) : The position of the central bright fringe will depend on angle θ, thickness of plate, distance between the slits and refractive index of the glass. For example, if $d \sin\theta = (\mu - 1)t$, the central bright fringe is at O. If $d \sin\theta > (\mu - 1)t$, the central bright frigne lies above O, and if $d \sin\theta < (\mu - 1)t$, the central bright fringe lies below O. Therefore the answer is (a).

13. (b) : $C_1 = 4\pi\varepsilon_0 b, C_2 = \frac{4\pi\varepsilon_0 ab}{b-a}$

$C_3 = \frac{4\pi\varepsilon_0 b^2}{b-a}, C_4 = 4\pi\varepsilon_0 b$

Clearly $C_1 = C_4$

also $C_3 > C_1 = C_4\ \left(\because\ \frac{b^2}{b-a} > b\right)$

Now $C_2 = \frac{4\pi\varepsilon_0 ab}{b-a} \approx 4\pi\varepsilon_0 a \Rightarrow C_3 > C_1 = C_4 > C_2$

14. (a) : The steel wire is first stretched by an object of specific gravity ρ in air. Then the object is half submerged in water. The stretching force diminishes due to upthrust of water on the object. Let σ denote specific gravity of water.

Weight of the object = $V\rho g$

Upthrust of water on object = $\frac{V}{2}\sigma g$.

$\therefore$ Tension $T' = V\rho g - \frac{V\sigma g}{2}$

or $T' = Vg\left(\frac{2\rho - \sigma}{2}\right)$

$\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ where $T = V\rho g$

$\upsilon' = \frac{1}{2l}\sqrt{\frac{T'}{\mu}}\ \therefore\ \frac{\upsilon'}{\upsilon} = \sqrt{\frac{T'}{T}}$

or $\frac{\upsilon'}{\upsilon} = \sqrt{\frac{Vg(2\rho - 1)}{2} \times \frac{1}{Vg\rho}}$

or $\upsilon' = \upsilon\sqrt{\frac{2\rho - 1}{2\rho}}$ or $\upsilon' = 300\left[\frac{2\rho - 1}{2\rho}\right]^{1/2}$.

15. (c) : Relative error in measurement of time,

$$\frac{\Delta t}{t} = \frac{1\text{ s}}{40\text{ s}} = \frac{1}{40}$$

Time period, $T = \frac{40\text{ s}}{20} = 2\text{ s}$

Error in measurement of time period,

$$\Delta T = T \times \frac{\Delta t}{t} = 2\text{ s} \times \frac{1}{40} = 0.05\text{ s}$$

The time period of simple pendulum is

$T = 2\pi\sqrt{\frac{l}{g}}$ or $T^2 = \frac{4\pi^2 l}{g}$ or $g = \frac{4\pi^2 l}{T^2}$

$\therefore\ \frac{\Delta g}{g} = \frac{2\Delta T}{T} = 2 \times \frac{1}{40} = \frac{1}{20}\left(\because\ \frac{\Delta T}{T} = \frac{\Delta t}{t}\right)$

Percentage error in determination of g is

$$\frac{\Delta g}{g} \times 100 = \frac{1}{20} \times 100 = 5\%$$

16. (c) : As intensity due to electric field

$$I_E = \frac{1}{2}c\varepsilon_0 E^2,$$

Intensity due to magnetic field $I_B = \frac{cB^2}{2\mu_0}$

$$\frac{I_E}{I_B} = \frac{\frac{1}{2}c\varepsilon_0 E^2}{cB^2/2\mu_0} = (\varepsilon_0\mu_0)(E/B)^2 = \left(\frac{1}{c^2}\right)(c^2) = 1$$

(as $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ and $E/B = c$)

17. (b) : Reynolds number,

$R_e = \frac{v\rho D}{\eta}$ and $v = \frac{4Q}{\pi D^2}$ (as $Q = \frac{\pi D^2}{4}v$),

$R_e = \frac{4Q\rho}{\pi D\eta}$

Here,

$D = 1.25\text{ cm} = 1.25 \times 10^{-2}\text{ m}$

$\rho = 10^3\text{ kg m}^{-3}, \eta = 10^{-3}\text{ Pa s}$

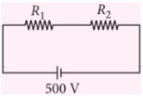
$Q = 5 \times 10^{-5}\text{ m}^3\text{ s}^{-1}$

$$R_e = \frac{4 \times 5 \times 10^{-5} \times 10^3}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} = 5095 \approx 5100.$$

For $R_e > 3000$, the flow is turbulent.

18. (c) : Let resistance of bulbs are R_1 and R_2

$\Rightarrow\ \frac{(200)^2}{R_1} = \frac{(300)^2}{R_2}$

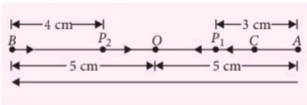


$\frac{R_2}{R_1} = \frac{9}{4}$

$$V_1 = IR_1 = \left(\frac{500}{R_1+R_2}\right)\times R_1;\ V_2 = \frac{500}{R_1+R_2}\times R_2$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9};\ \frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^2 \times \frac{R_2}{R_1} = \frac{4}{9}$$

19. (b): (a) Particle at P_1 is 3 cm away from A and is going towards B. As such sign of v is positive. Force and acceleration, which are directed towards O, also have positive signs.



(b) Particle at C is going from A to B and as such sign of v is positive.

(c) Particle at P_2 is 4 cm away from B and is going towards O. As such sign of v is negative. Force and acceleration which are directed towards O also have negative signs.

(d) Particle at B is going towards O and as such signs of acceleration and force are negative.

20. (c) : In β-emission, a neutron of nucleus decays into a photon and a β-particle.

$${}^1_0n \rightarrow {}^1_1\text{H} + {}^{\;\,0}_{-1}e$$

21. (0): $a = \dfrac{m_2 g - m_1 g \sin\theta}{m_1+m_2} = \dfrac{(4g - 8g\sin 30°)}{m_1+m_2} = 0$

22. (1.72) : Comparing $B = B_0 \sin(kz - \omega t)$ T with $B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T,
we get $B_0 = 12 \times 10^{-8}$ T
Average intensity of the beam,

$$I_v = \frac{cB_0^2}{2\mu_0} = \frac{3\times10^8 \times (12\times10^{-8})^2}{2\times4\pi\times10^{-7}} = 1.72\ \text{W m}^{-2}$$

23. (6.3) : $\dfrac{X}{R_0} = \dfrac{l}{100-l}$

Since null point remains unchanged

$$\frac{X}{R_0} = \frac{40}{60};\ R_0 = 6\ \Omega$$

And $6 = \dfrac{100R_t}{R_t+100}$; $R_t = 6.38\ \Omega$

$$\therefore\quad \alpha = \frac{R_t - R_0}{R_0 \Delta t} = 6.3\times10^{-4}\ \text{K}^{-1}$$

24 (0.02) : In the given circuit, diode D_1 is forward biased while D_2 is reverse biased. Moreover, D_1 has a forward resistance of 50 Ω.

$$\therefore\quad I = \frac{6}{50+150+100} = \frac{6}{300} = 0.02\ \text{A}$$

Current through 100 Ω resistance = 0.02 A.

25. (19) : Efficiency of a perfect engine working between – 3°C and 27°C (*i.e.*, T_2 = 270 K and T_1 = 300 K)

$$\eta_{\text{engine}} = 1 - \frac{T_2}{T_1} = 1 - \frac{270\ \text{K}}{300\ \text{K}} = 0.1$$

Since efficiency of the refrigerator ($\eta_{\text{ref.}}$) is 50% of η_{engine}

$$\therefore\quad \eta_{\text{ref.}} = 0.5\ \eta_{\text{engine}} = 0.05$$

If Q_1 is the heat transferred per second at higher temperature by doing work W, then

$$\eta_{\text{ref.}} = \frac{W}{Q_1} \text{ or } Q_1 = \frac{W}{\eta_{\text{ref.}}} = \frac{1\ \text{kJ}}{0.05} = 20\ \text{kJ}$$

(as W = 1 kW × 1s = 1 kJ)

Since $\eta_{\text{ref.}}$ is 0.05, heat removed from the refrigerator per second, *i.e.*,

$$Q_2 = Q_1 - \eta_{\text{ref.}} Q_1 = Q_1(1 - \eta_{\text{ref.}})$$
$$= 20\ \text{kJ}\ (1 - 0.05) = 19\ \text{kJ}$$

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BRAIN MAP

ELECTROMAGNETIC INDUCTION

Michael Faraday

A magnetic field can produce an electric field that can drive a current. This link between a magnetic field and the electric field is now known as Faraday's law of induction. The observations by Michael Faraday and other scientists which led to this law were at first just basic science. Today, however, applications of this basic science is everywhere.

MAGNETIC FLUX AND FARADAY'S LAW

- Magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$
- Faraday's Laws : It states that whenever magnetic flux linked with a coil changes, an emf is induced in the coil.

Induced emf, $\varepsilon = -N\dfrac{d\phi_B}{dt}$

Induced current, $I = \dfrac{\varepsilon}{R} = N\dfrac{(-d\phi_B/dt)}{R}$

Induced charge flow, $\Delta Q = I\Delta t = -N\dfrac{\Delta\phi_B}{R}$

- The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that has produced it.

A wire moving in B produces emf

Motional emf

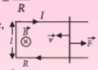
- On a straight conducting wire, $\varepsilon = Bvl$
- On a rotating conducting wire about one end $\varepsilon = \dfrac{B\omega l^2}{2}$

Here, $\vec{B}, \vec{v}(= \omega r\hat{v})$ and $\vec{l}$ are perpendicular to each other.

Quantitative idea behind motional emf

Energy Consideration in Motional emf

- Emf in the wire = Bvl
- Induced current, $I = \dfrac{\varepsilon}{R} = \dfrac{Bvl}{R}$
- Force exerted on the wire, $F = \dfrac{B^2l^2v}{R}$



- Power required to move the wire $P = \dfrac{B^2l^2v^2}{R}$

It is dissipated as Joule heat.

Special features of induced electric field

Induced Electric Field

- It is produced by change in magnetic field in a region. This is non-conservative in nature.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} = -A\frac{dB}{dt} \neq 0$$

- This is also known as integral form of Faraday's law.

Change in flux produces eddy current in a conducting plate

Eddy Current

- The currents induced in surface of bulk pieces of conductors when the magnetic flux linked with the conductor changes are known as Eddy currents. Slotting the conductor reduces the eddy current.
- Applications : Electromagnetic damping, Induction furnace, Electric power meter, Magnetic braking in trains.

Direction of induced emf/current is given by Lenz's law

Lenz's Law

- The direction of the induced current is such that it opposes the change that has induced it.
- If a current is induced by an increasing(decreasing) flux, it will weaken (strengthen) the original flux.
- It is a consequence of the law of conservation of energy.

Rotation of coil in B produces motional emf

Electric Generator

- Mechanical energy is converted into electrical energy by virtue of electromagnetic induction.
- Induced emf, $\varepsilon = NAB\omega \sin\omega t = \varepsilon_0 \sin\omega t$
- Current, $I = \dfrac{NBA\omega}{R}\sin\omega t = I_0 \sin\omega t$

Rate of change of current in the coil induces emf

Inductance

- Emf induced in the coil/conductor, $\varepsilon = -L\dfrac{dI}{dt}$
- Coefficient of self induction $L = \dfrac{N}{I}\phi_B = \dfrac{-\varepsilon}{dI/dt}$
- Self inductance of a coil $L = \dfrac{1}{2}\mu_0\pi N^2 R$
- Self inductance of a long solenoid $L = \mu_0\mu_r n^2 Al = \dfrac{\mu_0\mu_r N^2 A}{l}$
- Mutual inductance, $M = \dfrac{N_2\phi_2}{I_1} = \dfrac{-\varepsilon_2}{(dI_1/dt)} = \dfrac{-\varepsilon_1}{(dI_2/dt)}$
- Mutual inductance of two closely wound circular coils, $M \propto N_1N_2$
- Mutual inductance of two long coaxial solenoids

$$M = \mu_0\mu_r\pi r_1^2 n_1 n_2 l = \frac{\mu_0\mu_r N_1 N_2 A_1}{l}$$

- Coefficient of coupling, $k = \dfrac{M}{\sqrt{L_1L_2}}$

For perfect coupling, $k = 1$ so, $M = \sqrt{L_1L_2}$

Energy stored in the inductor (magnetic field)

Magnetic Energy

- Energy stored in an inductor $U_B = \dfrac{1}{2}LI^2$
- Energy stored in the solenoid, $U_B = \dfrac{1}{2\mu_0}B^2Al$
- Magnetic energy density, $u_B = \dfrac{U_B}{V} = \dfrac{B^2}{2\mu_0}$

Required inductance can be achieved by

Combination of Inductors

- Inductors in series, $L_S = L_1 + L_2 \pm 2M$
- Inductors in parallel, $L_P = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$
- If coils are far away, then $M = 0$

So, $L_S = L_1 + L_2$ and $L_P = \dfrac{L_1L_2}{L_1 + L_2}$

Growth/decay of current in an inductor

L – R Circuit

- Current growth in L–R circuit $I = I_0(1 - e^{-t/\tau_L})$
- Current decay in L–R circuit $I = I_0(e^{-t/\tau_L})$

Here, τ_L = Time constant = $\dfrac{L}{R}$

$I_0 = \dfrac{\varepsilon}{R}$

JEE 2020 PRACTICE PAPER ADVANCED

Exam on 23rd August

PAPER-I

SECTION 1 (Maximum Marks : 12)

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. ONLY ONE of these four options is the correct answer.*
- *For each question, choose the correct option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

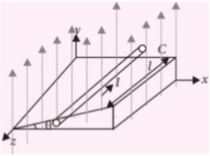
Full Marks : +3 If ONLY the correct option is chosen.
Zero Marks : 0 If none of the options is chosen i.e. the question is unanswered).
Negative Marks : –1 In all other cases.

1. Consider the earth as a uniform sphere of mass M and radius R. Imagine a straight smooth tunnel made through the earth which connects any two points on its surface. The time taken by a particle to go from one end to other through the tunnel is

(a) $2\pi\sqrt{\frac{R^3}{GM}}$ (b) $\pi\sqrt{\frac{R^3}{GM}}$

(c) $\sqrt{\frac{R^3}{GM}}$ (d) $\sqrt{\frac{R^3}{2GM}}$

2. A conducting wire of length l and mass m is placed on two inclined rails as shown in the figure. A current I is flowing in the wire in the direction shown. When no magnetic field is present in the region, the wire is just on the verge of sliding. When a vertically upward magnetic field is switched on, the wire starts moving up the incline. The distance travelled by the wire as a function of time t will be



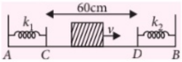
(a) $\frac{1}{2}\left[\frac{IBl}{m}-2g\right]t^2$

(b) $\frac{1}{2}\left[\frac{IBl}{m}\times\frac{1}{\cos\theta}-2g\sin\theta\right]t^2$

(c) $\frac{1}{2}\left[\frac{IBl}{m}-2g\sin\theta\right]t^2$

(d) $\frac{1}{2}\left[\frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta}-2g\sin\theta\right]t^2$

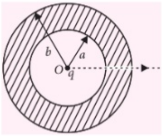
3. Two light springs of force constants k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one ends of each spring is fixed on rigid supports and the other end is free as shown in the figure.



The distance CD between the free ends of the springs is 60 cm. If the block moves along AB with a velocity 120 cm s^{-1} in between the springs, calculate the period of oscillation of the block.
(k_1 = 1.8 N m^{-1}, k_2 = 3.2 N m^{-1}, m = 200 g)

(a) 3 s (b) 4 s (c) 2.84 s (d) 4.35 s

4. A point charge q is located at centre O of a spherical uncharged conducting layer provided with a small orifice as shown in the figure. The inside and outside radii of the layer are equal to a and b respectively. What amount of work has to be performed to slowly transfer the charge q from the point O through the orifice and into infinity ?



(a) $\frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$ (b) $\frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$

(c) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$ (d) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$

SECTION 2 (Maximum Marks : 32)

- *This section contains EIGHT (08) questions.*
- *Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

  Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.
- *For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option (i.e. the question is unanswered) will get 0 marks ; and choosing any other combination of options will get –1 mark.*

5. For a certain radioactive substance, it is observed that after 4 h, only 6.25% of the original sample is left undecayed. It follows that

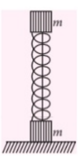
(a) the half-life of the sample is 1 h

(b) the mean life of the sample is $\frac{1}{\ln 2}$ h

(c) the decay constant of the sample is $\ln(2)\ \text{h}^{-1}$

(d) after a further 4 h, the amount of the substance left over would by only 0.39% of the original amount.

6. A system consists of two identical cubes, each of mass m, linked together by the compressed weightless spring of stiffness k. The cubes are also connected by a thread which is burned through at a certain moment.



(a) The lower cube will bounce up after the thread has been burned through when the initial compression of the spring is $\frac{2mg}{k}$.

(b) If the initial compression of the spring is $7\ mg/k$, then centre of gravity of this system will rise to height $\frac{8mg}{k}$.

(c) The lower cube will bounce up after the thread has been burned through when the initial compression of the spring is $\frac{5mg}{k}$.

(d) All are correct.

7. It is desired to make a long cylindrical conductor whose temperature coefficient of resistivity at 20°C will be close to zero. If such a conductor is made by assembling alternate disks of iron and carbon, find the ratio of the thickness of a carbon disk to that an iron disk.

(For carbon, $\rho = 3500 \times 10^{-8}\ \Omega$ m and $\alpha = -0.50 \times 10^{-3}\ °C^{-1}$ for iron, $\rho = 9.68 \times 10^{-8}\ \Omega$ m and $\alpha = 6.5 \times 10^{-3}\ °C^{-1}$)

(a) 0.36 (b) 0.036 (c) 1.0 (d) 2.0

8. A rocket set for vertical firing weighs 50 kg and contains 450 kg of fuel. It can have a maximum exhaust velocity of 2 km s^{-1}.

(a) Minimum rate of fuel consumption to just lift it off the launching pad is 2.45 kg s^{-1}.

(b) Minimum rate of fuel consumption to give it an acceleration of 20 m s^{-2} is 3.5 kg s^{-1}.

(c) The speed of the rocket is 4.2 km s^{-1} when the rate of consumption of fuel is 10 kg s^{-1} after whole of the fuel is consumed.

(d) All are correct.

9. One mole of a diatomic ideal gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A. The process $A \rightarrow B$ is an adiabatic compression, $B \rightarrow C$ is isobaric expansion, $C \rightarrow D$ is an adiabatic expansion, and $D \rightarrow A$ is isochoric. The volume ratios are $V_A/V_B = 16$ and $V_C/V_B = 2$ and the temperature at A is $T_A = 300$ K.

(a) Temperature of the gas at B is 909 K.

(b) Temperature of the gas at D is 791 K.

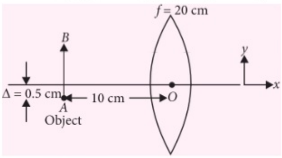
(c) The efficiency of the cycle is 61.4%.

(d) The efficiency of the cycle is 38.6%.

10. A 30 cm violin string with linear mass density 0.652 g m^{-1} is placed near a loudspeaker that is fed by an audio oscillator variable frequency. It is found that the string is set into oscillation only at the frequencies 880 Hz and 1320 Hz as the frequency of the oscillator is varied continuously over the range 500-1500 Hz. What is the tension in the string ?

(a) 120 N (b) 60 N (c) 90.8 N (d) 45.4 N

11. A linear object of size 1.5 cm is placed at 10 cm from a lens of focal length 20 cm. The optic centre of lens and the object are displaced a distance Δ. The magnification of the image formed is m. (Take optic centre as origin). The coordinates of image of A and B are (x_1, y_1) and (x_2, y_2) respectively. Then



(a) $(x_1, y_1) = (-20\text{ cm}, -1\text{ cm})$
(b) $(x_2, y_2) = (-20\text{ cm}, 2\text{ cm})$
(c) $m = 3$ (d) $m = 2$

12. For a certain metal, the K absorption edge is at 0.172 Å. The wavelength of K_α, K_β and K_γ lines of K series are 0.210 Å, 0.192 Å, and 0.180 Å, respectively. The energies of K, L and M orbits are E_K, E_L and E_M, respectively. Then

(a) $E_K = -13.07$ keV (b) $E_L = -7.52$ keV
(c) $E_M = -3.21$ keV (d) $E_K = 13.04$ keV

SECTION 3 (Maximum Marks : 18)

- ***This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks : +3 ***If ONLY the correct numerical value is entered.***

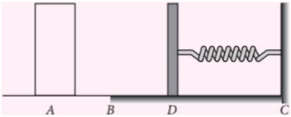
Zero Marks : 0 ***In all other cases.***

13. A current I flows in a rectangularly shaped wire whose center lies at $(x_0, 0, 0)$ and whose vertices are located at the points $A(x_0 + d, -a, -b)$, $B(x_0 - d, a, -b)$, $C(x_0 - d, a, b)$, and $D(x_0 + d, -a, b)$ respectively. Assume that $a, b, d << x_0$. Find the magnitude of magnetic dipole moment vector of the rectangular wire frame in J T^{-1}. (Given: $b = 10$ m, $d = 4$ m, $a = 3$ m, $I = 0.01$ A)

14. A very long, straight, thin wire carries -3.60 nC m^{-1} of fixed negative charge. The wire is to be surrounded by a uniform cylinder of positive charge, radius 1.50 cm, coaxial with the wire. The volume charge density ρ of the cylinder is to be selected so that the net electric field outside the cylinder is zero. Calculate the required positive charge density ρ (in μC m^{-3}).

15. A long coaxial cable consists of two thin-walled conducting cylinders with inner radius 2 cm and outer radius 8 cm. The inner cylinder carries a steady current 0.1 A, and the outer cylinder provides the return path for that current. The current produces a magnetic field between the two cylinders. Find the energy stored in the magnetic field for length 5 m of the cable. Express answer in nJ (use ln 2 = 0.7).

16. A 0.5 kg block slides from the point A on a horizontal track with an initial speed 3 m s^{-1} towards a weightless horizontal spring of length 1 m and force constant 2 N m^{-1}. The part AB of the track is frictionless and the part BC has the coefficient of static and kinetic friction as 0.22 and 0.20 respectively. If the distance AB and BD are 2 m and 2.14 m respectively, find the total distance (nearest integer) in m through which the block moves before it comes to rest completely. [$g = 10$ m s^{-2}]



17. An open organ pipe containing air resonates in fundamental mode due to a tuning fork. The measured values of length l (in cm) of the pipe and radius r (in cm) of the pipe are $l = 94 \pm 0.1$, $r = 5 \pm 0.05$. The velocity of the sound in air is accurately known. The maximum percentage error in the measurement of the frequency of that tuning fork by this experiment is given by α^2%. Find the value of 10 α.

18. In a certain polytropic process, the volume of argon was increased four times. Simultaneously, the pressure decreased eight times. Find the molar heat capacity (in SI unit) of argon in this process, assuming the gas to be ideal.

SECTION 1 (Maximum Marks : 32)

- *This section contains EIGHT (08) questions.*
- *Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer (s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

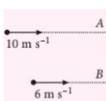
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –2 In all other cases.

- *For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option (i.e. the question is unanswered) will get 0 marks ; and choosing any other combination of options will get –1 mark.*

1. A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length $\sqrt{3}$ m.



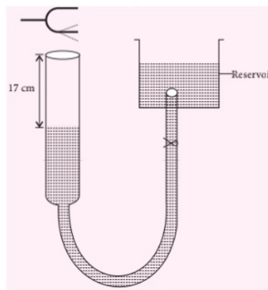
Two particles, each of mass 0.08 kg, are moving on the same surface and towards the bar in a direction perpendicular to the bar, one with a velocity of 10 m s^{-1}, and other with 6 m s^{-1} as shown in the figure. The first particle strikes the bar at point A and the other at point B. Points A and B are at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. The loss of the kinetic energy of the system in the collision process is

(a) 2 J (b) 4 J
(c) 2.72 J (d) 5.44 J

2. n drops of a liquid each with surface energy E join to form a single drop. Then

(a) some energy will be released in the process
(b) some energy will be absorbed in the process
(c) the energy released will be $E(n - n^{2/3})$
(d) the energy absorbed will be $nE(2^{2/3} - 1)$

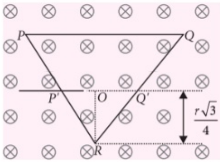
3. A tuning fork vibrating with a frequency of 512 Hz is kept close to the open end of a tube filled with water, as shown in the figure.



The water level in the tube is gradually lowered. When the water level is 17 cm below the open end, maximum intensity of sound is heard. The room temperature is 20°C. Then

(a) Speed of sound in air at room temperature is 360 m s^{-1}.
(b) Speed of sound in air at 0°C is 336 m s^{-1}.
(c) If the water in the tube is replaced with mercury, then the intensity of reflected sound increases.
(d) If the water in the tube is replaced with mercury, then the intensity of reflected sound decreases.

4. PQR is an equilateral triangular frame of mass m and side r. It is at rest under the action of horizontal magnetic field B as shown in the figure and the gravitational field.



(a) The frame remains at rest if the current in the frame is $\frac{2mg}{rB}$.

(b) The frame remains at rest if the current in the frame is $\frac{2mg}{rB\sqrt{3}}$.

(c) The frame is in simple harmonic motion when frame is slightly displaced in its plane perpendicular to PQ. The period of oscillation is $\pi\left[\frac{r\sqrt{3}}{g}\right]^{1/2}$

(d) For same as in above option, the period of oscillation is $\pi\left[\frac{3r}{2g}\right]^{1/2}$

5. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ. The height of the liquid in one vessel is h_1 and in other vessel is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the two vessels are connected ?

(a) $\frac{\rho A g}{4}(h_1 - h_2)^2$ (b) $\frac{\rho A g}{4}(h_1 + h_2)^2$

(c) $\frac{\rho A g}{2}(h_1 - h_2)^2$ (d) $\frac{\rho A g}{2}(h_1 + h_2)^2$

6. A particle of mass m moves in a certain plane due to a force F whose vector rotates in that place with a constant angular velocity ω. Assuming the particle to be stationary at the moment $t = 0$, then

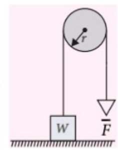
(a) its velocity as a function of time is $\left(\frac{F}{m\omega}\right)\sin\left(\frac{\omega t}{2}\right)$

(b) its velocity as a function of time is $\left(\frac{2F}{m\omega}\right)\sin\left(\frac{\omega t}{2}\right)$

(c) the distance covered by the particle between two successive stops is $\frac{8F}{m\omega^2}$

(d) the mean velocity over two successive stops is $\frac{4F}{\pi m\omega}$.

7. A massless rope is tossed over a wooden dowel of radius r in order to lift a heavy object of weight W off the floor, as shown in the figure. The coefficient of sliding friction between the rope and the dowel is μ. Which of the following relation is correct for minimum downward pull (F_{min}) on the rope necessary to lift the object ?



(a) $F_{min} = We^{-\pi\mu}$ (b) $F_{min} = We^{+\pi\mu}$

(c) $F_{min} = -We^{\pi\mu}$ (d) $F_{min} = We^{-\pi}$

8. If dimensions of length are expressed as $G^x c^y h^z$, where G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

(a) $x = \frac{1}{2}, y = \frac{1}{2}$

(b) $x = \frac{1}{2}, z = \frac{1}{2}$

(c) $y = -\frac{3}{2}, z = \frac{1}{2}$

(d) $y = \frac{1}{2}, z = \frac{3}{2}$

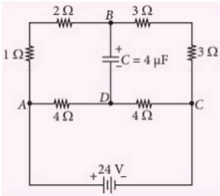
SECTION 2 (Maximum Marks : 18)

- *This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : *+3 If ONLY the correct numerical value is entered.*

Zero Marks : *0 In all other cases.*

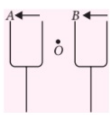
9. For the given circuit in the steady state condition, charge on the capacitor is $q_0 = 16\ \mu C$. If now the battery is removed and the nodes A and C are shorted. The time during which charge on the capacitor becomes $4\ \mu C$ is $t(\mu s)$ and emf of battery is $\varepsilon(V)$. Find the value of $\frac{3t}{\varepsilon \ln 2}$.



10. A glass of refractive index 1.5 is coated with a thin layer of thickness t of refractive index 1.8. Light of wavelength λ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. If $\lambda = 648$ nm, obtain the least value of t(in 10^{-8} m) for which the rays interfere constructively.

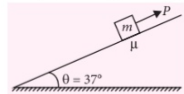
11. A charged particle enters a uniform magnetic field with velocity $v_0 = 4$ m s^{-1} perpendicular to it, the length of magnetic field is $x = \frac{\sqrt{3}}{2}R$, where R is the radius of the circular path of the particle in the field. Find the magnitude of change in velocity (in m s^{-1}) of the particle when it comes out of the field.

12. Two tuning forks A and B each of natural frequency 85 Hz move with velocity 10 m s^{-1} relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him as shown in the figure. A wind with a speed 10 m s^{-1} is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m s^{-1}]



13. In two calorimeters, we poured 200 g of water each at temperatures of +30 °C and +40 °C. From the hot calorimeter 50 g of water, is poured into cold calorimeter and stirred. Then from cold calorimeter 50 g of water is poured in hot and again stirred. How many times do you have to pour the same portion of water back and forth so that the temperature difference between water in the calorimeters becomes less than 3 °C ? Heat loss during the transfer and head capacity of calorimeters is neglected.

14. A block of mass m is being pulled up the rough incline, inclined at an angle 37° with horizontal by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. Find the maximum speed (in m s^{-1}) of the block during the course of ascent.
[Take: P = 60 W, m = 1 kg, μ = 0.5]



Section 3 (Maximum Marks : 12)

- *This section contains FOUR (04) match the columns type questions.*
- *Each question has FOUR options. ONLY ONE of these four options corresponds to the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*
- *Full Marks : +3 If ONLY the correct option is chosen.*
- *Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).*
- *Negative Marks : −1 In all other cases.*

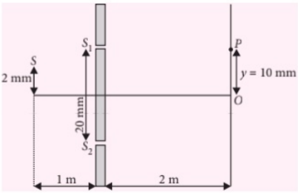
15. In each situation of Column-I, a physical quantity related to orbiting electron in hydrogen-like atom is given. The terms Z and n given in Column-II have usual meaning in Bohr's theory. Match the quantities in Column-I with the terms they depend on in Column-II.

Column-I		Column-II	
(A)	Frequency of orbiting electron	(P)	is directly proportional to Z^2
(B)	Angular momentum of orbiting electron	(Q)	is directly proportional to n
(C)	Magnetic moment of orbiting electron	(R)	is inversely proportional to n^3
(D)	The average current due to orbiting of electron	(S)	is independent of Z

Code:

(a) A → (P, Q), B → (R, S), C → (Q, S), D → (P, S)
(b) A → (P, R), B → (Q, S), C → (Q, S), D → (P, R)
(c) A → (R, Q), B → (Q, P), C → (Q, R), D → (P, Q)
(d) A → (Q, S), B → (Q, P), C → (P, R), D → (R, S)

16. In Young's double-slit experiment, the point source *S* is placed slightly off the central axis as shown in the figure. If $\lambda = 500$ nm, then match the following.



Column-I		Column-II	
(A)	Nature and order of interference at point *P*, *OP* = 10 mm	(P)	Bright fringe of order 80
(B)	Nature and order of interference at point *O*	(Q)	Bright fringe of order 262
(C)	If a transparent paper (refractive index μ = 1.45) of thickness t = 0.02 mm is pasted on S_1, *i.e.*, one of the slits, the nature and order of the interference at *P*	(R)	Bright fringe of order 62
(D)	After inserting the transparent paper in front of slit S_1, the nature and order of interference at *O*	(S)	Bright fringe of order 280

Code:

(a) A → (P), B → (Q), C → (R), D → (S)
(b) A → (Q), B → (P), C → (S), D → (R)
(c) A → (R), B → (Q), C → (P), D → (S)
(d) A → (S), B → (P), C → (Q), D → (R)

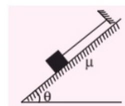
17. In the following, Column-I lists some physical quantities and the Column-II gives approximate energy values associated with some of them. Choose the appropriate value of energy from Column-II for each of the physical quantities in Column-I.

Column-I		Column-II	
(A)	Energy of thermal neutrons	(P)	0.025 eV
(B)	Energy of X-rays	(Q)	8 MeV
(C)	Binding energy per nucleon	(R)	3 eV
(D)	Photoelectric threshold of a metal	(S)	10 keV

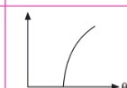
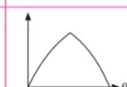
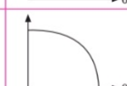
Code:

(a) A → (S), B → (Q), C → (R), D → (P)
(b) A → (R), B → (P), C → (S), D → (Q)
(c) A → (P), B → (S), C → (Q), D → (R)
(d) A → (Q), B → (S), C → (P), D → (R)

18. A block of mass *m* is put on a rough inclined plane of inclination θ, and is tied with a light thread shown. Inclination θ is increased gradually from $\theta = 0°$ to $\theta = 90°$.



Match the column according to corresponding curves.

Column-I		Column-II	
(A)	Tension in the thread versus θ	(P)	
(B)	Normal reaction between the block and the incline versus θ	(Q)	
(C)	Friction force between the block and the incline versus θ	(R)	
(D)	Net interaction force between the block and the incline versus θ	(S)	

Code:

(a) A → (Q), B → (S), C → (R), D → (P)
(b) A → (R), B → (P), C → (Q), D → (S)
(c) A → (Q), B → (R), C → (S), D → (P)
(d) A → (R), B → (S), C → (P), D → (Q)

PAPER-I

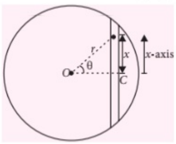
1. **(b) :** Suppose at some instant the particle is at radial distance r from centre of earth O. Since, the particle is constrained to move along the tunnel, we define its position as distance x from C. Hence, equation of motion of the particle is,

$$ma_x = F_x$$

The gravitational force on mass m at distance r is,

$$F = \frac{GMmr}{R^3} \quad \text{(towards } O\text{)}$$

Therefore, $F_x = -F\sin\theta = -\frac{GMmr}{R^3}\left(\frac{x}{r}\right) = -\frac{GMm}{R^3}\cdot x$



Since, $F_x \propto -x$, motion is simple harmonic in nature. Further,

$$ma_x = -\frac{GMm}{R^3}\cdot x \text{ or } a_x = -\frac{GM}{R^3}\cdot x$$

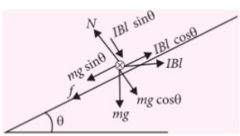
∴ Time period of oscillation is,

$$T = 2\pi\sqrt{\left|\frac{x}{a_x}\right|} = 2\pi\sqrt{\frac{R^3}{GM}}$$

The time taken by particle to go from one end to the other is $\frac{T}{2}$.

$$\therefore \quad t = \frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$$

2. **(d) :** The front view of the arrangement is shown in figure.



From initial condition, $mg\sin\theta = \mu mg\cos\theta$

$\Rightarrow \mu = \tan\theta$

$$ma = IBl\cos\theta - mg\sin\theta - \mu N$$

$$N = mg\cos\theta + IBl\sin\theta$$

$$\Rightarrow a = \frac{IBl}{m}\cos\theta - 2g\sin\theta - \frac{IBl}{m}\frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta$$

Now, $s = \frac{1}{2}at^2 = \frac{1}{2}\left[\frac{IBl}{m}\frac{\cos 2\theta}{\cos\theta} - 2g\sin\theta\right]t^2$

3. **(c) :** The block moves along AB with a velocity of 120 cm s^{-1}, in between the springs. Since AB is a smooth table, the journey from D to C or from C to D is performed with the uniform speed of 120 cm s^{-1}. There is neither an acceleration nor a retardation in this region.

The block moves to right and compresses the spring along DB. The spring offers restoring force and the block comes back to D. Thus half of oscillation is completed in this journey. Let the time be t_1.

For a spring, $T = 2\pi\sqrt{\frac{m}{k}}$

$$\therefore \quad t_1 = \frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{0.2}{3.2}} = 0.785 \text{ s}$$

Now, t_2 = Time to travel from D to C = $\frac{\text{Distance } DC}{\text{velocity}}$

or $\quad t_2 = \frac{60}{120} = 0.5$ s

Similarly t_3 = Time taken at spring (C to A)

or $\quad t_3 = \frac{T'}{2} = \frac{2\pi}{2}\sqrt{\frac{0.2}{1.8}} = 1.05$ s

Again t_4 = Time to travel from C to D

$$t_4 = \frac{60}{120} = 0.5 \text{ s}$$

∴ Total time taken to complete one oscillation = T
$= t_1 + t_2 + t_3 + t_4$

Time period = 0.785 + 0.5 + 1.05 + 0.5

∴ Period of oscillation = 2.84 s approximately.

4. **(a) :** Initially, there will be induced charges of magnitude $-q$ and $+q$ on the inner and outer surface of the spherical layer respectively. Hence, the total electrical energy of the system is the sum of self energies of spherical shells, having radii a and b, and their mutual energies including the point charge q.

$$U_i = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0 b} + \frac{1}{2}\frac{(-q)^2}{4\pi\varepsilon_0 a} + \frac{-qq}{4\pi\varepsilon_0 a} + \frac{qq}{4\pi\varepsilon_0 b} + \frac{-qq}{4\pi\varepsilon_0 b}$$

or, $\quad U_i = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{a}\right]$

Finally, charge q is at infinity hence, $U_f = 0$

Now, work done by the agent = increment in the energy

$$= U_f - U_i = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

5. **(a, b, c, d) :** We have, 6.25% = $\frac{6.25}{100}=\frac{1}{16}=\frac{1}{2^4}$

The given time of 4 h thus equals 4 half-lives so the half-life is 1 h.

Since half-life = $\frac{\ln 2}{\text{decay constant}}$

and mean life = $\frac{1}{\text{decay constant}}$

decay constant, $\lambda = \ln 2\ h^{-1}$

mean life, $\tau = \frac{1}{\ln 2}$ h

After further 4 h, the amount left over would be $\frac{1}{2^4}\times\frac{1}{2^4}$, i.e., $\frac{1}{256}$ or $\frac{100}{256}$% or 0.39% of original amount.

6. **(b, c) :** The initial compression in the spring (Δl) must be such that after burning of the thread, the upper cube rises to a height that produces a tension in the spring that is atleast equal to the weight of the lower cube. Actually, the spring will first go from its compressed state to its natural length and then get elongated beyond this natural length. Let l be the maximum elongation produced under these circumstances.

Then $kl = mg$ ...(i)

Now, from energy conservation,

$$\frac{1}{2}k\Delta l^2 = mg(\Delta l + l) + \frac{1}{2}kl^2 \quad \text{...(ii)}$$

(Because at maximum elongation of the spring, the speed of upper cube becomes zero)

From equations (i) and (ii),

$$\Delta l^2 - \frac{2mg\Delta l}{k} - \frac{3m^2g^2}{k^2} = 0 \text{ or, } \Delta l = \frac{3mg}{k}, \frac{-mg}{k}$$

Therefore, acceptable solution of Δl equals $\frac{3mg}{k}$.

Let v be the velocity of upper cube at the position (say, at C) when the lower block breaks off the floor, then from energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta l^2 - l^2) - mg(l + \Delta l) \left(\because \Delta l = \frac{7mg}{k}\right)$$

or, $v^2 = 32\frac{mg^2}{k}$

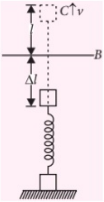
At the position C, the velocity of CM, $v_c = \frac{mv+0}{2m} = \frac{v}{2}$.

Let the CM of the system (spring + two cubes) further rises up to Δy_{c_2}.

Now, from energy conservation,

$$\frac{1}{2}(2m)v_c^2 = (2m)g\,\Delta y_{c_2}$$

or, $\Delta y_{c_2} = \frac{v_c^2}{2g} = \frac{v^2}{8g} = \frac{4mg}{k}$



But, uptil position C, the C.M. of the system has already elevated by,

$$\Delta y_{c1} = \frac{(\Delta l + l)m + 0}{2m} = \frac{4mg}{k}$$

Hence, the net displacement of the C.M. of the system, in upward direction

$$\Delta y_c = \Delta y_{c_1} + \Delta y_{c_2} = \frac{8mg}{k}$$

7. **(b) :** Change in the resistance of the conductor on increasing its temperature,

$$R - R_0 = R_0\alpha(T - T_0).$$

The disks will be effectively in series, so we will add the resistances to get the total. Looking only at one pair of disk, we have

$R_c + R_i = R_{0c}(\alpha_c(T - T_0) + 1) + R_{0i}(\alpha_i(T - T_0) + 1)$
$= R_{0c} + R_{0i} + (R_{0c}\alpha_c + R_{0i}\alpha_i)(T - T_0).$

This equation will only be constant if the coefficient for the term $(T - T_0)$ vanishes.

Then $R_{0c}\alpha_c + R_{0i}\alpha_i = 0$,

but $R = \rho L/A$, and the disks have the same cross sectional area, so

$$L_c\rho_c\alpha_c + L_i\rho_i\alpha_i = 0$$

or $\frac{L_c}{L_i} = -\frac{\rho_i\alpha_i}{\rho_c\alpha_c} = -\frac{9.68\times10^{-8}\times6.5\times10^{-3}}{3500\times10^{-8}\times(-0.50\times10^{-3})}$
$= 0.036$

8. **(a, c) :** To just lift off the rocket the launching pad, weight = thrust force

or $mg = v_r\left(\frac{-dm}{dt}\right)$ or $\left(\frac{-dm}{dt}\right) = \frac{mg}{v_r}$

Substituting the values, we get

$$\left(\frac{-dm}{dt}\right) = \frac{(450+50)(9.8)}{2\times10^3} = 2.45 \text{ kg s}^{-1}$$

Net acceleration, $a = 20$ m s^{-2}

$\therefore \quad ma = F_t - mg$ or $a = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$

This gives $\left(-\frac{dm}{dt}\right) = \frac{m(g+a)}{v_r}$

Substituting the values, we get

$$\left(-\frac{dm}{dt}\right) = \frac{(450+50)(9.8+20)}{2\times10^3} = 7.45 \text{ kg s}^{-1}$$

The rate of fuel consumption is 10 kg s^{-1}. So, the time for the consumption of entire fuel is

$$t = \frac{450}{10} = 45 \text{ s}$$

Speed of the rocket after $t = 45$ s

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

Here, $u = 0$, $v_r = 2 \times 10^3$ m s^{-1}, $m_0 = 500$ kg and $m = 50$ kg

Substituting the values, we get

$$v = 0 - (9.8)(45) + (2 \times 10^3) \ln\left(\frac{500}{50}\right)$$

or $v = -441 + 4605.17$

or $v = 4164.17$ m s^{-1} $= 4.164$ km s^{-1} ≈ 4.2 km s^{-1}

9. (a, b, c) : Given: $n = 1$ mole

$\gamma = 1.4$ for diatomic gas.

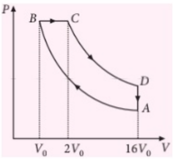
$$T_A = 300 \text{ K}, \frac{V_A}{V_B} = 16, \frac{V_C}{V_B} = 2$$

$A \rightarrow B$: adiabatic compression

$B \rightarrow C$: isobaric expansion

$C \rightarrow D$: adiabatic expansion

$D \rightarrow A$: isochoric process.



To find T_B

Process $A \rightarrow B$ is adiabatic

$$\therefore \left(\frac{T_B}{T_A}\right) = \left(\frac{V_A}{V_B}\right)^{\gamma-1}$$

or $\frac{T_B}{T_A} = (16)^{1.4-1}$

$= (16)^{2/5} = (256)^{1/5} = 3.03$

$\therefore$ $T_B = T_A \times 3.03$ or $T_B = 300 \times 3.03$

or $T_B = 909$ K ...(i)

To find T_D

$B \rightarrow C$ is an isobaric process.

$$\therefore \frac{V_B}{T_B} = \frac{V_C}{T_C} \quad \text{or} \quad T_C = T_B\left(\frac{V_C}{V_B}\right)$$

or $T_C = 909 \times 2$ or $T_C = 1818$ K ...(ii)

$C \rightarrow D$ is an adiabatic process.

$$\therefore \frac{T_D}{T_C} = \left(\frac{V_C}{V_D}\right)^{\gamma-1} \text{ or } T_D = T_C \times \left(\frac{V_C}{V_D}\right)^{\gamma-1} \quad (\because V_D = V_A)$$

or $T_D = 1818 \times \left(\frac{2}{16}\right)^{1.4-1} = 1818\left(\frac{1}{8}\right)^{2/5}$

or $T_D = 1818 \times 0.435$ or $T_D = 791.4$ K ...(iii)

To find efficiency of the cycle.

$$\% \text{ efficiency } (\eta) = \frac{\text{Net work done}}{\text{Heat absorbed}} \times 100\%$$

or $\eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$

where Q_2 = Heat released in the cycle

Q_1 = Heat absorbed in the cycle

For adiabatic process AB and CD, $\Delta Q = 0$

or $Q_{AB} = Q_{CD} = 0$

Now $Q_{DA} = nC_V\, dT$ or $Q_{DA} = (1)\left(\frac{5R}{2}\right)(T_A - T_D)$

$$Q_2 = \left(\frac{5 \times 8.31}{2}\right)(300 - 791.4) = -10208.8 \text{ J}$$

Negative sign shows that heat is lost by the system

Again $Q_{BC} = nC_p\, \Delta T$

$$Q_1 = (1)\left(\frac{7R}{2}\right)(T_C - T_B)$$

$$Q_1 = 1 \times \left(\frac{7 \times 8.31}{2}\right) \times (1818 - 909)$$

$$Q_1 = \frac{7 \times 8.31 \times 909}{2} = 26438.3 \text{ J}$$

$$\therefore \eta = \left(1 - \frac{Q_2}{Q_1}\right)100\%$$

or $\eta = \left(1 - \frac{10208.8}{26438.3}\right)100\%$ or $\eta = 61.4\%$

Hence $T_B = 909$ K, $T_D = 791.4$ K ≈ 791 K

$\eta = 61.4\%$

10. (d) : A 30 cm string fixed at both ends will resonate if the frequency is given by $\upsilon = nv/2L$, where n is an integer, L the length of the string, and v the wave-speed on the string. The string is observed to resonate at 880 Hz and then again at 1320 Hz, so the two corresponding values of n must differ by 1. We can then write two equations

$$(880 \text{ Hz}) = \frac{nv}{2L} \text{ and } (1320 \text{ Hz}) = \frac{(n+1)v}{2L}$$

or, $\frac{(880 \text{ Hz})}{n} = \frac{v}{2L}$ and $\frac{(1320 \text{ Hz})}{n+1} = \frac{v}{2L}$.

Combining these two equations, we get

$$\frac{(880 \text{ Hz})}{n} = \frac{(1320 \text{ Hz})}{n+1},$$

$$(n + 1)(880 \text{ Hz}) = n(1320 \text{ Hz}),$$

$$n = \frac{(880 \text{ Hz})}{(1320 \text{ Hz}) - (880 \text{ Hz})} = 2.$$

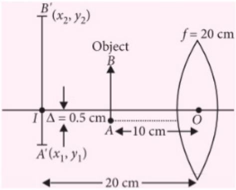
For a string fixed at both ends (or a pipe closed or opened at both ends), the frequency difference between two adjacent harmonics is the same as the fundamental frequency, *i.e.*, $\upsilon_1 = \upsilon_{n+1} - \upsilon_n$

$\therefore \quad v = 2(0.3)\dfrac{(880)}{2} = 264 \text{ m s}^{-1}$

The tension, in the string

$T = \mu v^2 = 0.652 \times 10^{-3} \times (264)^2 = 45.4$ N

11. (a, b, d): $u = -10$ cm, $f = 20$ cm



$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$; $\dfrac{1}{v} = \dfrac{1}{20} - \dfrac{1}{10} = \dfrac{1-2}{20} = \dfrac{-1}{20} \Rightarrow v = -20$ cm

Magnification, $m = \dfrac{v}{u} = \dfrac{-20}{-10} = 2$

Height of image = 1.5 × 2 = 3 cm

The y_1 coordinate of a point A on the image will be $y_1 = (-0.5) \times 2 = -1$ cm

The y_2 coordinate of a point B on the image will be $y_2 = 1 \times 2 = 2$ cm

12. (a, b, c) : Energy of K absorption edge

$E = \dfrac{1242 \text{ eVnm}}{0.0172 \text{ nm}} = 72.21 \times 10^3 \text{ eV} = 72.21 \text{ keV}$

Energy of K_α line is

$E_{K_\alpha} = \dfrac{hc}{\lambda_\alpha} = \dfrac{1242 \text{ eVnm}}{0.021 \text{ nm}} = 59.14 \text{ keV}$

Similarly, $E_{K_\beta} = \dfrac{1242}{0.0192} = 64.69 \text{ keV}$

$E_{K_\gamma} = \dfrac{1242}{0.0180} = 69 \text{ keV}$

Energy of K shell = $(E_{K_\alpha} - E)$

= (59.14 – 72.21) KeV = – 13.07 keV

Energy of L shell = $E_{K_\beta} - E$

= 64.69 keV – 72.21 keV = – 7.52 keV

Energy of M shell = $E_{K_\gamma} - E$

= 69 keV – 72.21 keV = –3.21 keV

13. (2) : Magnetic moment of a current carrying loop,

$\vec{\mu} = I\vec{S}$

Area of the loop, $\vec{S} = \overline{AB} \times \overline{BC}$

Here, $\overline{AB} = -2d\,\hat{i} + 2a\,\hat{j}$, $\overline{BC} = 2b\,\hat{k}$

$\therefore \quad \vec{S} = (-2d\,\hat{i} + 2a\,\hat{j}) \times (2b\,\hat{k}) = 4bd\,\hat{j} + 4ab\,\hat{i}$

$\therefore \quad |\vec{\mu}| = I|\vec{S}| = 4Ib\sqrt{a^2 + d^2}$

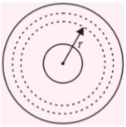
$= 4 \times 0.01 \times 10 \times \sqrt{3^2 + 4^2}$

$= 0.4 \times 5 = 2 \text{ J T}^{-1}$

14. (5) : We don't really need to write an integral, we just need the charge per unit length in the cylinder to be equal to zero. This means that the positive charge in cylinder must be +3.60 nC m^{-1}. This positive charge is uniformly distributed in a circle of radius R = 1.50 cm, so

$$\rho = \frac{3.60 \text{ nC m}^{-1}}{\pi R^2} = \frac{3.60 \text{ nC m}^{-1}}{\pi (0.015 \text{ m})^2} \approx 5 \,\mu\text{C m}^{-3}.$$

15. (7) : The magnetic field inside is only due to the current of the inner cylinder. $B = \dfrac{\mu_0 i}{2\pi r}$



Magnetic field energy density is not uniform in the space between the cylinders. At a distance r from the centre

$$u_B = \frac{B^2}{2\mu_0} = \frac{\mu_0 i^2}{8\pi^2 r^2}$$

Energy in volume of element (length l)

$$dU_B = u_B\,dV = \frac{\mu_0 i^2}{8\pi^2 r^2}(2\pi r l)dr = \frac{\mu_0 i^2 l}{4\pi}\frac{dr}{r}$$

$$U_B = \frac{\mu_0 i^2 l}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 l}{4\pi}\ln\frac{b}{a}$$

Using values, we get U_B = 7 nJ

16. (4) : As the track AB is frictionless, the block moves this distance without loss in its initial KE = $\dfrac{1}{2}mv^2$ = $\dfrac{1}{2} \times 0.5 \times 3^2 = 2.25$ J.

In the path BD as friction is present, so work done against friction

= $\mu_k mg \times (BD) = 0.2 \times 0.5 \times 10 \times 2.14 = 2.14$ J

So, at D the KE of the block = 2.25 – 2.14 = 0.11 J.

Now, if the spring is compressed by x

$$0.11 = \frac{1}{2} \times k \times x^2 + \mu_k mgx$$

or $\quad 0.11 = \dfrac{1}{2} \times 2 \times x^2 + 0.2 \times 0.5 \times 10x$

or $\quad x^2 + x - 0.11 = 0$

On solving, x = 0.1 m or –1.1 m

$x \neq -1.1$ m, so, x = 0.1 m

After moving the distance $x = 0.1$ m the block comes to rest. Now the compressed spring exerts a force:

$$F = kx = 2 \times 0.1 = 0.2 \text{ N}$$

on the block while limiting frictional force between block and track is $f_L = \mu_s mg = 0.22 \times 0.5 \times 10 = 1.1$ N. Since, $F < f_L$. The block will not move back. So, the total distance moved by the block

$$= AB + BD + 0.1 = 2 + 2.14 + 0.1 = 4.24 \text{ m}$$

17. (4) : $\upsilon = \dfrac{v}{2(l+2e)}$

$$\therefore \quad \upsilon = \frac{v}{2(l+2\times 0.6r)} = \frac{v}{2(l+1.2r)}$$

$$\therefore \quad \frac{\Delta\upsilon}{\upsilon} = \frac{\Delta v}{v} - \frac{\Delta(l+1.2r)}{l+1.2r} = \frac{\Delta v}{v} - \frac{\Delta l + 1.2\Delta r}{l+1.2r}$$

Here $\dfrac{\Delta v}{v} = 0$; $\dfrac{\Delta\upsilon}{\upsilon}\times 100 = -\dfrac{\Delta l + 1.2\Delta r}{l+1.2r}\times 100\%$

For maximum % error: $\Delta l = 0.1$ cm, $\Delta r = 0.05$ cm

$$\left(\frac{\Delta\upsilon}{\upsilon}\times 100\%\right)_{max} = \frac{0.1 + 1.2\times 0.05}{94 + 1.2\times 5}\times 100\%$$

$$= 0.16\% = \alpha^2\%$$

$\therefore \quad \alpha = 0.4$. Hence, $10\alpha = 4$.

18. (4) : Let the process is polytropic. According to the law pV^n = constant

Thus, $p_f V_f^n = p_i V_i^n$ or, $\left(\dfrac{V_f}{V_i}\right)^n = \dfrac{p_i}{p_f}$

Given $\dfrac{V_f}{V_i} = \alpha = 4$ and $\dfrac{p_i}{p_f} = \beta = 8$

So, $\alpha^n = \beta$ or $\ln\beta = n\ln\alpha$ or $n = \dfrac{\ln\beta}{\ln\alpha}$

In the polytropic process, molar heat capacity is given by

$$C_n = \frac{R(n-\gamma)}{(n-1)(\gamma-1)} = \frac{R}{\gamma-1} - \frac{R}{n-1}$$

$$= \frac{R}{\gamma-1} - \frac{R\ln\alpha}{\ln\beta - \ln\alpha}$$

So, $C_n = \dfrac{8.314}{1.4-1} - \dfrac{8.314\ln 4}{\ln 8 - \ln 4} = 4.16 \approx 4 \text{ J mol}^{-1}\text{K}^{-1}$

PAPER-II

1. (c)

2. (a, c) : Surface tension = Surface energy per unit area

r = Radius of each small drop

R = Radius of big drop

$$n\times\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \text{ or } R = n^{\frac{1}{3}} r$$

Initial surface energy $= E_i = n \times 4\pi r^2 \times T = nE$

Final surface energy $= E_f = 4\pi R^2 \times T = 4\pi r^2 n^{2/3}\cdot T$

$$E_f = n^{2/3}E$$

Energy released $= E_i - E_f = E(n - n^{2/3})$

3. (b, c) : The tube filled with water behaves as a closed organ pipe of length, $L = 17$ cm $= 0.17$ m

$$\text{As } \upsilon = \frac{v_{20}}{4L}$$

$$v_{20} = 4\upsilon L = 4\times 512\times 0.17 = 348.16 \text{ m s}^{-1}$$

$$\text{As } \frac{v_{20}}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+20}{273}}$$

$$v_0 = 348.16\times\sqrt{\frac{273}{293}} = 336 \text{ m s}^{-1}$$

The resonance will still be observed for 17cm length of air column above mercury. However, due to more complete reflection of sound waves at mercury surface, the intensity of reflected sound increases.

4. (a, c): According to Lenz's law, current I is in clockwise direction, in the loop.

$$P'Q' = 2P'O = \frac{2}{\tan 60^\circ}\left(\frac{r\sqrt{3}}{4}\right) = \frac{r}{2}$$

For equilibrium, $mg = I\left(\dfrac{r}{2}\right)B$

$$I = \frac{2mg}{rB}$$

If loop is displaced by x,

F = Restoring force

$$= -I\left[\left(\frac{2}{\tan 60^\circ}\right)\left(\frac{r\sqrt{3}}{4} + x\right)\right]B + mg$$

$$= -\frac{IrB}{2} + mg - \frac{2IB}{\sqrt{3}}x = -\frac{2IB}{\sqrt{3}}.x$$

$F \propto -x$ *i.e.*, motion is SHM.

$$a = -\frac{2IB}{m\sqrt{3}}x$$

$$\therefore \quad \text{Time period, } T = 2\pi\sqrt{\frac{m\sqrt{3}}{2IB}} = \pi\left[\frac{r\sqrt{3}}{g}\right]^{\frac{1}{2}}$$

5. (a)

6. (b, c, d) : Let us fix the x-y co-ordinate system to the given plane, taking x-axis in the direction along which the force vector was oriented at the moment $t = 0$, then the fundamental equation of dynamics expressed via the projection on x and y-axis gives at latter time t,

$$F\cos\omega t = m\frac{dv_x}{dt} \quad \text{...(i)}$$

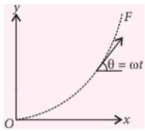
$$\text{and } F\sin\omega t = m\frac{dv_y}{dt} \quad \text{...(ii)}$$

From (i), $\int_0^{v_x} dv_x = \frac{F}{m}\int_0^t \cos\omega t\, dt$ ...(iii)

or $v_x = \frac{F}{m\omega}\sin\omega t$

and similarly from (ii), $v_y = \frac{F}{m\omega}(1 - \cos\omega t)$ ...(iv)

Hence, $v = \sqrt{v_x^2 + v_y^2} = \left(\frac{2F}{m\omega}\right)\sin\left(\frac{\omega t}{2}\right)$



It is seen from this that the velocity v turns into zero after the time interval Δt, which can be found from the relation,

$\omega\frac{\Delta t}{2} = \pi$. Consequently, the sought distance, is

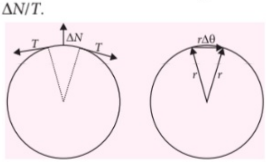
$$s = \int_0^{\Delta t} v\, dt = \frac{2F}{m\omega}\int_0^{2\pi/\omega}\sin\left(\frac{\omega t}{2}\right)dt = \frac{8F}{m\omega^2}$$

Average velocity,

$$<v> = \frac{\int v\, dt}{\int dt} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{2F}{m\omega}\sin\left(\frac{\omega t}{2}\right)dt = \frac{4F}{\pi m\omega}$$

7. (b) : The rope wraps around the dowel and there is a contribution to the frictional force Δf from each small segment of the rope where it touches the dowel. There is also a normal force ΔN at each point where the contact occurs.

In the figure we can form a triangle with long side T and short side ΔN. In another we see a triangle with long side r and short side $r\Delta\theta$. These triangles are similar, so $r\Delta\theta/r = \Delta N/T$.



Now $\Delta f = \mu\Delta N$ and $T(\theta) + \Delta f \approx T(\theta + \Delta\theta)$. Combining, and taking the limit as $\Delta\theta \to 0$, $dT = df$

$$\int\frac{1}{\mu}\frac{dT}{T} = \int d\theta$$

Integrating both sides of this expression,

$$\int_{T_1}^{T_2}\frac{1}{\mu}\frac{dT}{T} = \int_0^{\pi} d\theta\ ;\ \frac{1}{\mu}[\ln T]_{T_1}^{T_2} = \pi$$

$$T_2 = T_1 e^{\pi\mu}.$$

In this case T_1 is the weight and T_2 is the downward force.

$\therefore\quad F = We^{\pi\mu}$

8. (b, c) : $[L] = [G^x c^y h^z]$ whence, $x = 1/2$, $y = -3/2$, $z = 1/2$.

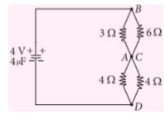
9. (4) : Potential difference across the capacitor is

$$\frac{q_0}{C} = \frac{16}{4} = 4\text{ V}$$

emf of the battery, $\varepsilon = 24$ V

Time constant, $\tau = RC = (4\ \Omega)(4\ \mu F) = 16\ \mu s$

Equation of discharge $q = q_0 e^{-t/\tau}$



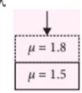
$$4\ \mu C = (16\ \mu C)\, e^{\left(-\frac{t}{16}\right)}$$

$$t = 32\ln 2\ \mu s$$

$$\therefore\quad \frac{3t}{\varepsilon\ln 2} = \frac{3\times 32\ln 2}{24\ln 2} = 4$$

10. (9) : Path difference between rays reflected from upper and lower faces of layer = $2\mu t\cos r = 2\mu t$ (for normal incidence). But there is abrupt change of $\lambda/2$ in path of light at upper surface. So actual path difference is $2\mu t - \lambda/2$

For constructive interference, $2\mu t - \frac{\lambda}{2} = n\lambda$

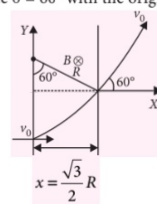


$t = \frac{(2n+1)\lambda}{4\mu}$. For least thickness $n = 0$.

$$\therefore\quad t_{min} = \frac{\lambda}{4\mu} = \frac{648}{4\times 1.8}\text{ nm} = 90\text{ nm}$$

$$= 9\times 10^{-8}\text{ m}$$

11. (4) : The particle will come out of the magnetic field at an angle $\theta = 60°$ with the original direction.



$$\Delta\vec{v} = (v_0\cos 60°\,\hat{i} + v_0\sin 60°\,\hat{j} - v_0\hat{i}) \Rightarrow |\Delta\vec{v}| = v_0 = 4\text{ m/s}$$

12. (5) : $\upsilon_A = \upsilon\left(\frac{v - v_m}{v - v_m + v_s}\right) = \frac{340 - 10}{340 - 10 + 10}\times 85 = \frac{330}{4}$Hz

$$\upsilon_B = \upsilon\left(\frac{v + v_m}{v + v_m - v_s}\right) = \frac{340 + 10}{340 + 10 - 10}\times 85 = \frac{350}{4}\text{Hz}$$

$\upsilon_{beat} = \upsilon_B - \upsilon_A = \frac{350}{4} - \frac{330}{4} = \frac{20}{4} = 5$ Hz

13. (5) : $200 \times S \times 30 + 50 \times S \times 40 = 250 \times S \times T_1$

$T_1 = 32\ °C$

$32 \times S \times 50 + 150 \times S \times 40 = 200 \times S \times T_2$

$T_2 = 38\ °C$

$200 \times S \times 32 + 50 \times S \times 38 = 250 \times S \times T_3$

$T_3 = 33.2\ °C$

$33.2 \times S \times 50 + 150 \times S \times 38 = 200 \times S \times T_4$

$T_4 = 36.8\ °C$

$36.8 \times S \times 50 + 200 \times S \times 33.2 = 250 \times S \times T_5$

$T_5 = 33.92\ °C$

14. (6) : Let the maximum speed be v.

$P = (mg\sin\theta + \mu\, mg\cos\theta)v$

$$\Rightarrow\ v = \frac{P}{mg\sin\theta + \mu mg\cos\theta}$$

$$= \frac{60}{1\times10\times\frac{3}{5} + \frac{1}{2}\times1\times10\times\frac{4}{5}} = 6\ \text{m s}^{-1}$$

15. (b)

A. $\upsilon = \frac{mZ^2e^4}{4\varepsilon_0^2h^3n^3} \Rightarrow \upsilon \propto \frac{Z^2}{n^3}$

B. $L = \frac{nh}{2\pi} \Rightarrow L \propto n$

C. Magnetic moment: $M = IA = \frac{e}{2\pi r} v\pi r^2$

$= \frac{e}{2}vr = \frac{e}{2m}(mvr) = \frac{e}{2m}L$

$\Rightarrow M = \frac{e}{2m}\left(\frac{nh}{2\pi}\right) \Rightarrow M \propto n$

D. $i = \frac{ev}{2\pi r} = \frac{e}{2\pi}\left(\frac{\pi mZe^2}{n^2h^2\varepsilon_0}\right)\frac{Ze^2}{2\varepsilon_0 nh} \Rightarrow i \propto \frac{Z^2}{n^3}$

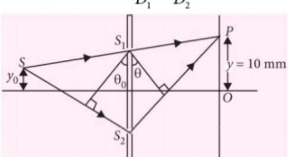
16. (d)

The optical path difference between the two waves arriving at P is

$\delta = (SS_2 + S_2P) - (SS_1 + S_1P)$

$= (SS_2 - SS_1) + (S_2P - S_1P)$

$= d\sin\theta_0 + d\sin\theta = \frac{dy_0}{D_1} + \frac{dy}{D_2}$.



$d = 20$ mm, $y_0 = 2$ mm, $D_2 = 2$ m, $D_1 = 1$ m, $y = 10$ mm

$\therefore\ \delta = \frac{20\times2}{1000} + \frac{20\times10}{2000} = 0.14$ mm

For a bright fringe, $\delta = n\lambda$

$\Rightarrow\ n = \frac{\delta}{\lambda} = \frac{0.14}{0.5\times10^{-3}} = 280$

At the origin O, $\delta' = \frac{dy_0}{D_1} = 0.04$ mm

$n' = \frac{\delta'}{\lambda} = \frac{0.04}{0.5\times10^{-3}} = 80$

Due to transparent paper, the change in optical path is

$(\mu - 1)t = (1.45 - 1)(0.02)$ mm $= 0.009$ mm

$\delta'' = 0.14$ mm $- 0.009$ mm $= 0.131$ mm

$\Rightarrow\ n = \frac{0.131}{0.5\times10^{-3}} = 262$

Due to transparent paper, the path difference at O,

$\delta''' = \delta' - (\mu - 1)t = (0.04 - 0.009)$ mm $= 0.031$ mm

$\Rightarrow\ n = \frac{0.031}{0.5\times10^{-3}} = 62$

17. (c) **18. (a)**

JEE Main

NUMERICAL VALUE TYPE QUESTIONS

Exam on 18^{th} - 23^{rd} July, 2020

1. Focal length of objective and eye piece of telescope are 200 cm and 4 cm respectively. The length (in cm) of telescope for normal adjustment is ____.

2. If the bulk modulus of water is 2100 MPa, the speed (in m s^{-1}) of sound in water is approximately____.

3. Distance between nodes on a string is 5 cm. Velocity of transverse wave is 2 m s^{-1}. Then the frequency (in Hz) is ______.

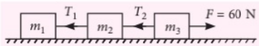
4. On moving a charge of 20 C by 2 cm, 2 J of work is done, then the potential difference (in V) between the points is ______.

5. A 5 ampere fuse wire can withstand a maximum power of 1 watt in the circuit. The resistance (in ohm) of the fuse wire is ______.

6. An ideal monoatomic gas is compressed adiabatically to $\left(\frac{1}{8}\right)^{th}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, then the value of a is ______.

7. A particle is projected from a horizontal plane with velocity of $5\sqrt{2}$ m s^{-1} at an angle. At highest point its velocity is found to be 5 m s^{-1}. Its range (in m) will be ______. (Take g = 10 m s^{-2})

8. Three blocks of masses m_1, m_2 and m_3 are connected by massless string as shown in the figure on a frictionless table. They are pulled with a force F = 60 N. If m_1 = 10 kg, m_2 = 20 kg and m_3 = 30 kg, then ratio $\frac{T_2}{T_1}$ is ______.



9. A fish at a depth of 12 cm in water is viewed by an observer on the bank of a lake. The height (in cm) up to which the image of fish is raised is ______. (Refractive index of lake water = 4/3)

10. When two progressive waves $y_1 = 4\sin(3x - 7t)$ and $y_2 = 3\sin(3x - 7t + \frac{\pi}{2})$ are superimposed, the amplitude of resultant wave is ______.

11. A Zener diode is specified as having a breakdown voltage of 9.1 V with a maximum power dissipation of 364 mW. The maximum current (in mA) the diode can handle is ______.

12. The angle of dip (in degree) at a certain place where the horizontal and vertical components of the earth's magnetic field are equal is ______.

13. A transformer is used to light a 100 W and 110 V lamp from a 220 V mains. If the main current is 0.5 A, the efficiency (in %) of the transformer is approximately ______.

14. A particle of mass 1 mg has the same wavelength as an electron moving with a velocity 3×10^6 m s^{-1}. The velocity of the particle is $x \times 10^{-18}$ m s^{-1}. The value of x is ______.
(Mass of electron = 9.1×10^{-31} kg)

15. The condition of apparent weightlessness can be created momentarily when a plane flies over the top of a vertical circle at a speed of 900 km h^{-1}. The radius (in km) of the vertical circle that the pilot must use is ______.

SOLUTIONS

1. **(204) :** In normal adjustment,
$L = f_o + f_e$ = 200 cm + 4 cm = 204 cm.

2. **(1450) :** Speed of sound in water is $v = \sqrt{\frac{B}{\rho}}$
where B is the bulk modulus and ρ is the density of water

$$v = \sqrt{\frac{2100\times10^6}{10^3}} \simeq 1450 \text{ m s}^{-1}$$

3. **(20) :** Distance between two nodes is $\frac{\lambda}{2}$.
$\frac{\lambda}{2}$ = 5 cm or λ = 10 cm = 0.1 m

Frequency, $\upsilon = \frac{v}{\lambda} = \frac{2}{0.1} = 20$ Hz

4. (0.1) : Potential difference between two points in an electric field is

$$V_A - V_B = \frac{W}{q_0}$$

where, W is work done by moving charge q_0 from point A to B. Here, $W = 2$ J, $q_0 = 20$ C

So, $V_A - V_B = \pm\frac{2}{20} = 0.1$ V

5. (0.04): $P = I^2R \Rightarrow R = \frac{P}{I^2} = \frac{1}{(5)^2} = 0.04\ \Omega$

6. (4): For an adiabatic process,

$TV^{\gamma-1}$ = constant ; $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

Substituting the given values, we get

$$T_iV_i^{\gamma-1} = aT_i\left(\frac{V_i}{8}\right)^{\gamma-1} \Rightarrow a = 8^{\gamma-1}$$

For a monoatomic gas, $\gamma = \frac{5}{3}$

$$\therefore \quad a = 8^{\frac{5}{3}-1} = 8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4.$$

7. (5): Here, velocity of projection, $u = 5\sqrt{2}$ m s^{-1}

At highest point velocity of projectile, $v = u\cos\theta$

$$\therefore \quad 5 = 5\sqrt{2}\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

$$\text{Range} = \frac{u^2\sin 2\theta}{g} = \frac{(5\sqrt{2})^2\sin 2\times 45°}{10} = \frac{25\times 2}{10} = 5.$$

8. (3): Here $m_1 = 10$ kg, $m_2 = 20$ kg, $m_3 = 30$ kg, $F = 60$ N . The common acceleration of the system is

$$a = \frac{F}{m_1+m_2+m_3} = \frac{60}{10+20+30} = 1 \text{ m s}^{-2}$$

$\therefore \quad T_1 = m_1a = 10a$

$T_2 = (m_1 + m_2)a = (10 + 20)a = 30a$

$$\frac{T_2}{T_1} = \frac{30a}{10a} = 3$$

9. (3): As $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

$$\text{Apparent depth} = \frac{12}{(4/3)} = 9 \text{ cm}$$

Height through which image of fish is raised = 12 – 9 = 3 cm.

10. (5): Here, $y_1 = 4\sin(3x - 7t), y_2 = 3\sin\left(3x - 7t + \frac{\pi}{2}\right)$

The phase difference between two waves is $\phi = \pi/2$

The amplitude of the resultant wave is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$= \sqrt{4^2 + 3^2 + 2\times 4\times 3\times\cos\frac{\pi}{2}} = 5 \quad \left(\because \cos\frac{\pi}{2} = 0\right)$$

11. (40): The maximum permissible current is

$$I_{Z_{max}} = \frac{P}{V_Z} = \frac{364\times 10^{-3}}{9.1} = 40 \text{ mA.}$$

12. (45): $\tan\delta = \frac{B_V}{B_H} = \frac{B_H}{B_H} = 1$ ($\because$ $B_H = B_V$ (Given))

$\therefore \quad \delta = 45°$.

13. (91): Output power, $P = 100$ W

Voltage across primary, $V_P = 220$ V

Current in the primary, $I_P = 0.5$ A

$$\text{Efficiency of a transformer, } \eta = \frac{\text{Output power}}{\text{Input power}}\times 100\%$$

$$= \frac{P}{V_PI_P}\times 100\% = \frac{100}{220\times 0.5}\times 100\% \approx 91\%.$$

14. (2.73): As, $\lambda = h/mv$; if for the two particles λ is same, then their momentum must be same.

$\therefore \quad m_pv_p = m_ev_e$ or $10^{-6}\times v_p = (9.1\times 10^{-31})\times(3\times 10^6)$

or $\quad v_p = \frac{(9.1\times 10^{-31})\times(3\times 10^6)}{10^{-6}} = 2.73\times 10^{-18}$ m s^{-1}

15. (6.4) : $v = 900$ km h^{-1} = $\frac{900\times 1000}{60\times 60} = 250$ m s^{-1};

$g = 9.8$ m s^{-2}

For apparent weightlessness, $\frac{mv^2}{r} = mg$

$$r = \frac{v^2}{g} = \frac{250\times 250}{9.8} = 6377.5 \text{ m} \approx 6.4 \text{ km}$$

PRACTICE PAPER

BITSAT

Exam date:
6^{th} to 10^{th}
August 2020

1. In a race for 100 m dash, the first and the second runners have a gap of one metre at the mid way stage. Assuming the first runner goes steady, by what percentage should the second runner increase his speed just to win the race?
   (a) 2% (b) 4%
   (c) more than 4% (d) less than 4%

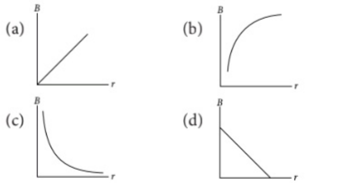
2. A sand bag of mass m is suspended from a long string. A bullet of mass $\frac{m}{20}$ moving with a horizontal velocity v strikes it and gets embedded into it. Calculate the velocity gained by the bag in this process and fraction of energy lost in the process.
   (a) $\frac{v}{21}; \frac{20}{21}$ (b) $\frac{v}{24}; \frac{18}{20}$
   (c) $\frac{v}{21}; \frac{20}{22}$ (d) $\frac{v}{19}; \frac{20}{21}$

3. The bodies situated on the surface of earth at its equator, become weightless, when the kinetic energy of rotation of earth about its axis is
   (a) MgR (b) $2MgR/5$
   (c) $MgR/5$ (d) $5MgR/2$

4. Two linear SHMs of equal amplitude A and angular frequencies ω and 2ω are impressed on a particle along the axes x and y respectively. If the initial phase difference between them is $\pi/2$, the resultant path followed by the particle is
   (a) $y^2 = x^2(1 - x^2/A^2)$ (b) $y^2 = 2x^2(1 - x^2/A^2)$
   (c) $y^2 = 4x^2(1 - x^2/A^2)$ (d) $y^2 = 8x^2(1 - x^2/A^2)$

5. A calorie is a unit of heat and equals 4.2 J. Suppose we employ a system of units in which the unit of mass is α kg, the unit of length is β m and the unit of time is γ s. In this new system, 1 calorie will be
   (a) $\alpha^{-1}\beta^{-2}\gamma^2$ (b) $4.2\alpha\beta^2\gamma^2$
   (c) $\alpha\beta^2\gamma^2$ (d) $4.2\alpha^{-1}\beta^{-2}\gamma^2$

6. Which of the following graph represents the variation of magnetic flux density B with distance r for a straight long wire carrying an electric current?

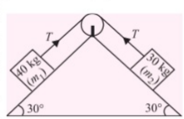


7. In an L – R circuit, the value of L is $(0.4/\pi)$ H and the value of R is 30 Ω. If in the circuit, an alternating emf of 200 V at 50 cycle per second is connected, the impedance of the circuit and current will be
   (a) 11.4 Ω, 17.5 A (b) 30.7 Ω, 6.5 A
   (c) 40.4 Ω, 5 A (d) 50 Ω, 4 A

8. A microscope has an objective of focal length 1.5 cm and eyepiece of focal length 2.5 cm. If the distance between objective and eyepiece is 25 cm, what is the approximate value of magnification produced for relaxed eye?
   (a) 75 (b) 110 (c) 140 (d) 25

9. Two small conducting spheres of equal radius have charges +10 μC and –20 μC and placed at a distance R from each other experience force F_1. If they are brought in contact and separated to the same distance, they experience force F_2. The ratio of F_1 to F_2 is
   (a) 1 : 2 (b) –8 : 1 (c) 1 : 8 (d) –2 : 1

10. Two masses 40 kg and 30 kg are connected by a weightless string passing over a frictionless pulley as shown in the figure. The tension in the string will be



   (a) 188 N (b) 368 N (c) 288 N (d) 168 N

11. At constant temperature, the volume of a gas is to be decreased by 4%. The pressure must be increased by
   (a) 5.34% (b) 4.16% (c) 2.96% (d) 3.86%

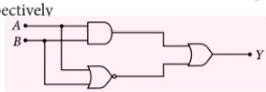
12. In a Young's double slit experiment, one of the slits is covered with a transparent sheet of thickness 3.6×10^{-5} m due to which position of central bright fringe shifts to a position originally occupied by 30^{th} fringe. The refractive index of the sheet, if $\lambda = 6000$ Å, is
(a) 1.5 (b) 1.9 (c) 1.3 (d) 1.7

13. At time $t = 0$, activity of a radioactive substance is 1600 Bq, at $t = 8$ s activity becomes 100 Bq. Find the activity at $t = 2$ s.
(a) 200 Bq (b) 400 Bq
(c) 600 Bq (d) 800 Bq

14. Find the ratio of minimum to maximum energy of radiation emitted by electron in ground state of Bohr's hydrogen atom.
(a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

15. In an experiment with sonometer, a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension in the string remains constant, the frequency of the second tuning fork is
(a) 204 Hz (b) 160 Hz (c) 400 Hz (d) 320 Hz

16. Refer to the arrangement of logic gates. For $A = 0$, $B = 0$ and $A = 1$, $B = 0$, the values of output Y are, respectively



(a) 0 and 1 (b) 1 and 0
(c) 1 and 1 (d) 0 and 0

17. If the length of stretched string is shortened by 40% and the tension is increased by 44 %, then the ratio of the final and initial fundamental frequencies is
(a) 2 : 1 (b) 3 : 2 (c) 3 : 4 (d) 1 : 3

18. A 220 V input is supplied to a transformer. The output circuit draws a current of 2.0 A at 440 V. If the efficiency of the transformer is 80%, the current drawn by the primary windings of the transformer is
(a) 3.6 A (b) 2.8 A (c) 2.5 A (d) 5.0 A

19. When a metal surface is illuminated with light of wavelength λ, the stopping potential is V_0. When the same surface is illuminated with light of wavelength 2λ, the stopping potential is $\frac{V_0}{4}$. If the velocity of light in air is c, the threshold frequency of photoelectric emission is
(a) $\frac{c}{6\lambda}$ (b) $\frac{c}{3\lambda}$ (c) $\frac{2c}{3\lambda}$ (d) $\frac{4c}{3\lambda}$

20. A launching vehicle carrying an artificial satellite of mass m is set for launch on the surface of the earth of mass M and radius R. If the satellite is intended to move in a circular orbit of radius $7R$, the minimum energy required to be spent by the launching vehicle on the satellite is (Gravitational constant = G)
(a) $\frac{GMm}{R}$ (b) $\frac{13GMm}{14R}$
(c) $\frac{GMm}{7R}$ (d) $\frac{GMm}{14R}$

21. An inductance coil is connected to an ac source through a 60 Ω resistance in series. The source voltage, voltage across the coil and voltage across the resistance are found to be 33 V, 27 V and 12 V respectively. Therefore, the resistance of the coil is
(a) 30 Ω (b) 45 Ω (c) 105 Ω (d) 75 Ω

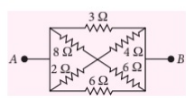
22. The cylindrical tube of spray pump has a cross-section of 8 cm^2, one end of which has 40 fine holes each of area 10^{-8} m^2. If the liquid flows inside the tube with a speed of 0.15 m min^{-1}, the speed with which the liquid is ejected through the holes is
(a) 50 m s^{-1} (b) 5 m s^{-1}
(c) 0.05 m s^{-1} (d) 0.5 m s^{-1}

23. A body is fired vertically upwards. At half the maximum height, the velocity of the body is 10 m s^{-1}. The maximum height raised by the body is (Take $g = 10$ m s^{-2})
(a) 5 m (b) 10 m (c) 15 m (d) 20 m

24. If the masses of deuterium and helium are 2.0140 amu and 4.0026 amu, respectively and 22.4 MeV energy is liberated in the reaction ${}^6_3Li + {}^2_1H \rightarrow {}^4_2He + {}^4_2He$, the mass of ${}^6_3Li$ is
(a) 6.015 amu (b) 6.068 amu
(c) 5.980 amu (d) 6.00 amu

25. The maximum range of projectile fired with some initial velocity is found to be 1000 m, in the absence of wind and air resistance. The maximum height reached by the projectile is
(a) 250 m (b) 500 m (c) 1000 m (d) 2000 m

26. In the adjoining figure, the equivalent resistance between A and B is



(a) $\left(\frac{17}{24}\right)\Omega$ (b) $\left(\frac{4}{3}\right)\Omega$
(c) $\left(\frac{24}{17}\right)\Omega$ (d) $\left(\frac{3}{4}\right)\Omega$

27. A satellite in a circular orbit of radius R has a period of 4 hours. Another satellite with orbital radius $3R$ around the same planet will have a period (in hours)

(a) 16 (b) 4 (c) $4\sqrt{27}$ (d) $4\sqrt{8}$

28. A body of mass M suspended from two springs separately executes simple harmonic motion. During oscillation the maximum velocity is equal in both cases. The ratio of amplitude $\frac{A_1}{A_2}$ is

(a) $\frac{k_1}{k_2}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1^2}{k_2^2}$

29. A uniform thin bar of mass $6m$ and length $12L$ is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is

(a) $20mL^2$ (b) $6mL^2$

(c) $\frac{12}{5}mL^2$ (d) $30mL^2$

30. An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})\text{m}$ to $\vec{r}_2 = (4\hat{i} + 6\hat{j})\text{m}$ under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})\text{N}$. The work done by this force is

(a) 63 J (b) 73 J (c) 83 J (d) 93 J

31. A charge Q is enclosed by a gaussian spherical surface of radius R. If the radius is doubled, then the outward electric flux will

(a) increase four times (b) be reduced to half

(c) remain the same (d) be doubled

32. A small bulb emits 100 W of electromagnetic radiation uniformly in all directions. What is the maximum energy density in the electric field at a point 1.0 m from the source?

(a) 10.5×10^{-8} J m^{-3} (b) 5.30×10^{-8} J m^{-3}

(c) 2.65×10^{-8} J m^{-3} (d) 20.1×10^{-8} J m^{-3}

33. In an interference pattern produced by two identical slits, the intensity at the site of the central maximum is I. The intensity at the same spot when either of the two slits is closed is I_0. Therefore,

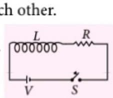
(a) $I = I_0$

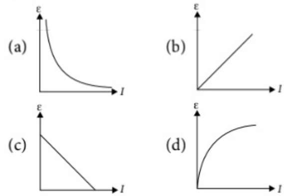
(b) $I = 2I_0$

(c) $I = 4I_0$

(d) I and I_0 are not related to each other.

34. In an L-R ciruit shown in figure, switch S is closed at time $t = 0$. If ε denotes the induced emf across inductor and I, the current in the circuit at any time t, then which of the following graphs, shows the variation of ε with I?

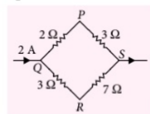




35. The time of vibration of a dip needle vibrating in the vertical plane in the magnetic meridian is 3 s. When the same magnetic needle is made to vibrate in the horizontal plane, the time of vibration is $3\sqrt{2}$ s. Then the angle of dip is

(a) 30° (b) 45° (c) 60° (d) 90°

36. A current of 2 A flows in the system of conductors as shown in the figure. The potential difference $V_P - V_R$ will be nearly



(a) – 2 V

(b) – 1 V

(c) + 1 V

(d) + 2 V

37. When the reverse potential in the semiconductor diode are 10 V and 15 V, the corresponding reverse currents are 10 μA and 35 μA respectively. The reverse resistance of junction diode will be

(a) 20 kΩ (b) 200 kΩ

(c) 2000 kΩ (d) none of these

38. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t °C, the linear stress developed in it is

(a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$ (c) $Y\alpha t$ (d) $\frac{1}{(Y\alpha t)}$

39. The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 ohm is applied. What is the resistance of the galvanometer?

(a) 12 Ω (b) 24 Ω (c) 36 Ω (d) 48 Ω

40. The angular momentum of an electron in the hydrogen atom is $\frac{3h}{2\pi}$. Here, h is Planck's constant. The kinetic energy of this electron is

(a) 4.35 eV (b) 1.51 eV

(c) 3.4 eV (d) 6.8 eV

SOLUTIONS

1. **(c)** : Let v_1, v_2 be the initial speeds of first and second runners. Let t be the time taken by them when the first runner has completed 50 m. During this time, the second runner has covered a distance $= 50 - 1 = 49$ m.

So, $t = \frac{50}{v_1} = \frac{49}{v_2}$ ...(i)

Suppose the second runner increases his speed to v_3 so that he covers the remaining distance (51 m) in time t. So

$$t = \frac{51}{v_3} = \frac{49}{v_2} \text{ or } v_3 = \frac{51}{49} \times v_2 = \left(1 + \frac{2}{49}\right) v_2$$

or $\frac{v_3}{v_2} - 1 = \frac{2}{49}$ or $\frac{v_3 - v_2}{v_2} = \frac{2}{49}$

or % increase $= \frac{2}{49} \times 100\% = 4.1\%$

i.e. more than 4%

2. **(a)** : Applying principle of conservation of linear momentum,

$$\frac{m}{20} \times v = \left(m + \frac{m}{20}\right) v' = \frac{21m}{20} v'$$

$$v' = \frac{v}{20} \times \frac{20}{21} = \frac{v}{21}$$

This is the velocity gained by the bag.

Initial kinetic energy of bullet, $E_1 = \frac{1}{2}\left(\frac{m}{20}\right) v^2$

Final kinetic energy of bag and bullet,

$$E_2 = \frac{1}{2}\left(m + \frac{m}{20}\right) v'^2 = \frac{1}{2}\left(\frac{21m}{20}\right)\left(\frac{v}{21}\right)^2$$

Loss of energy

$$= E_1 - E_2 = \frac{1}{2}\left(\frac{m}{20}\right) v^2 \left[1 - \frac{1}{21}\right] = \frac{1}{2}\left(\frac{m}{20}\right) v^2 \times \frac{20}{21}$$

Fraction of energy lost $= \frac{E_1 - E_2}{E_1} = \frac{20}{21}$

3. **(c)** : When there is a weightlessness in the body at the equator, then $g' = g - R\omega^2 = 0$

or $\omega = \sqrt{g/R}$

and linear velocity $\omega R = (\sqrt{g/R})R = \sqrt{gR}$

$\therefore$ Kinetic energy of rotation of earth $= \frac{1}{2} I\omega^2$

$$= \frac{1}{2} \times \frac{2}{5} MR^2 \times \omega^2 = \frac{1}{5} M(\omega R)^2 = \frac{1}{5} MgR$$

4. **(c)** : $x = A \sin(\omega t + \pi/2) = A \cos \omega t$

$\therefore$ $\cos \omega t = x/A$ and $\sin \omega t = \sqrt{1 - (x^2/A^2)}$

$y = A \sin 2\omega t = 2A \sin \omega t \cos \omega t$

or $y^2 = 4A^2 \sin^2 \omega t \cos^2 \omega t$

$$= 4A^2 \times \frac{x^2}{A^2} \times \left(\frac{A^2 - x^2}{A^2}\right) = 4x^2\left(1 - \frac{x^2}{A^2}\right)$$

5. **(d)**: 1 calorie = 4.2 J $\therefore$ [calorie] = $[ML^2T^{-2}]$.

Comparing with general dimensional formula $[M^aL^bT^c]$, we get

$a = 1, b = 2, c = -2$

$$\because \quad n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$\Rightarrow \quad n_2 = 4.2 \left[\frac{1 \text{ kg}}{\alpha \text{ kg}}\right]^1 \left[\frac{1 \text{ m}}{\beta \text{ m}}\right]^2 \left[\frac{1 \text{ s}}{\gamma \text{ s}}\right]^{-2} = 4.2\, \alpha^{-1}\beta^{-2}\gamma^2$$

6. **(c)** : Magnetic field induction at a point due to a long current carrying wire is related with distance r by relation $B \propto 1/r$. Therefore graph (c) is correct.

7. **(d)**: Here, $X_L = \omega L = 2\pi\upsilon L$

$$= 2\pi \times 50 \times \frac{0.4}{\pi} = 40\ \Omega$$

$R = 30\ \Omega$

$$\therefore \quad Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50\ \Omega$$

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{200}{50} = 4 \text{ A}$$

8. **(c)** : Length of the tube is $L = v_0 + f_e$

$v_0 = L - f_e = 25 - 2.5 = 22.5$ cm

Now applying $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$, we have

$$\frac{1}{22.5} - \frac{1}{u_0} = \frac{1}{1.5}$$

$\therefore$ $|u_0| \approx 1.6$ cm

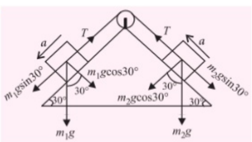
$$\therefore \quad |M| = \frac{v_0}{u_0} \times \frac{D}{f_e} = \left(\frac{22.5}{1.6}\right)\left(\frac{25}{2.5}\right) = 140$$

9. **(b)**: Here, $F_1 = \frac{k(+10)(-20)}{R^2} = \frac{-k \times 200}{R^2}$

As spheres are of equal radius, their capacities are same. On touching, the net charge = + 10 – 20 = –10 μC is shared equally between them *i.e*, each sphere carries –5 μC charge.

$$F_2 = \frac{k(-5)(-5)}{R^2} = \frac{k \times 25}{R^2} \quad \therefore \quad \frac{F_1}{F_2} = \frac{-8}{1}$$

10. **(d)**:



$m_1 g\sin30° - T = m_1 a$ ... (i)

$T - m_2 g\sin30° = m_2 a$ ... (ii)

Adding (i) and (ii),

$$a = \frac{m_1 g\sin 30° - m_2 g \sin 30°}{m_1 + m_2}$$

From eqn. (ii), we get

$$T = m_2 g \sin30° + m_2 a$$

$$= m_2 g\sin 30° + \frac{m_2}{m_1+m_2}(m_1 g\sin 30° - m_2 g\sin 30°)$$

$$= \frac{2m_1 m_2 g\sin 30°}{m_1+m_2} = \frac{2\times40\times30\times9.8\times(1/2)}{40+30}$$

$$= \frac{1200}{70}\times 9.8 = 168 \text{ N}$$

11. (b): At constant temperature, pV = constant

$$p_1V_1 = p_2V_2 \quad \text{or} \quad \frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Fractional change in volume

$$\frac{V_1 - V_2}{V_1} = \frac{4}{100} = \frac{1}{25}$$

$$1 - \frac{V_2}{V_1} = \frac{1}{25} \quad \text{or,} \quad \frac{V_2}{V_1} = \frac{24}{25}$$

$$\therefore \quad \frac{p_1}{p_2} = \frac{V_2}{V_1} = \frac{24}{25} \quad \text{or} \quad \frac{p_2}{p_1} = \frac{25}{24}$$

$$\frac{p_2 - p_1}{p_1} = \frac{25}{24} - 1 = \frac{1}{24}$$

% increase in pressure = $\frac{100}{24}$ = 4.16 %

12. (a): The position of 30th bright fringe $y_{30} = \frac{30\lambda D}{d}$

New position of central fringe is $y_0 = \frac{30\lambda D}{d}$

But we know , y_0 = shift due to transparent sheet

$= \frac{D}{d}(\mu - 1)t$

So, $\frac{30\lambda D}{d} = \frac{D}{d}(\mu - 1)t$

$$(\mu - 1) = \frac{30\lambda}{t} = \frac{30\times(6000\times10^{-10})}{(3.6\times10^{-5})} = 0.5$$

$\therefore \quad \mu = 1.5$

13. (d): Activity, $R = R_0\left(\frac{1}{2}\right)^n$

where n is the number of half-lives.

At $t = 8$ s, $100 = 1600\left(\frac{1}{2}\right)^n$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^n \quad \text{or} \quad n = 4$$

four half-lives are equivalent to 8 s. Hence, 2 s is equal to one half-life. So, in one half-life activity will fall half of 1600 Bq *i.e.*, 800 Bq.

14. (c): Energy of radiation corresponding to transition between two energy levels n_1 and n_2 is given by

$$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{eV}.$$

E is minimum when $n_1 = 1$ and $n_2 = 2$.

Hence, $E_{min} = 13.6\left(\frac{1}{1} - \frac{1}{4}\right)\text{eV} = 13.6\times\frac{3}{4}$ eV

E is maximum when $n_1 = 1$ and $n_2 = \infty$ (when the atom is ionised).

Hence

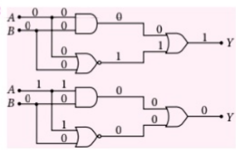
$$E_{max} = 13.6\left(1 - \frac{1}{\infty}\right) = 13.6 \text{ eV} \quad \therefore \quad \frac{E_{min}}{E_{max}} = \frac{3}{4}$$

15. (c): For sonometer, frequency of vibration of string, provided its tension and mass per unit length remain same, is given by $\upsilon \propto \frac{1}{l}$

$$\therefore \quad \frac{\upsilon_2}{\upsilon_1} = \frac{l_1}{l_2} \quad \text{or} \quad \frac{\upsilon_2}{256} = \frac{25}{16}$$

$$\upsilon_2 = \frac{256\times25}{16} = 400 \text{ Hz}$$

16. (b):



17. (a): Initial fundamental frequency of a stretched string is

$$\upsilon = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad \text{...(i)}$$

where the symbols have their usual meanings.

When the length of a stretched string is shortened by 40% and the tension is increased by 44%, then its length and tension become

$$L' = L - \frac{40}{100}L = \frac{3}{5}L,\ T' = T + \frac{44}{100}T = \frac{36}{25}T$$

Then, final fundamental frequency is

$$\upsilon' = \frac{1}{2L'}\sqrt{\frac{T'}{\mu}} = \frac{1}{2\left(\frac{3}{5}L\right)}\sqrt{\frac{36}{25}\frac{T}{\mu}} = \frac{2}{2L}\sqrt{\frac{T}{\mu}} \quad \text{...(ii)}$$

Dividing (ii) by (i), we get $\frac{\upsilon'}{\upsilon} = \frac{2}{1}$

18. (d): Here, $V_p = 220$ V, $I_s = 2$ A, $V_s = 440$ V

$\eta = 80\%$, $I_p = ?$

$$\eta = \frac{V_s I_s}{V_p I_p} \text{ or } I_p = \frac{V_s I_s}{\eta V_p}$$

Substituting the given values, we get

$$I_p = \frac{440 \times 2}{\frac{80}{100} \times 220} = 5 \text{ A}$$

19. (b)

20. (b): The energy of the satellite on the surface of the earth is

$$E_s = \text{KE} + \text{PE} = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$$

The energy of the satellite in an orbit of radius r is

$$E_o = \frac{1}{2}mv_o^2 + \left(-\frac{GMm}{r}\right)$$

$$= \frac{1}{2}m\left(\frac{GM}{r}\right) - \frac{GMm}{r} \quad \left[\text{As } v_o = \sqrt{\frac{GM}{r}}\right]$$

$$= -\frac{GMm}{2r}$$

The minimum energy required to be spent by the vehicle is

$$\Delta E = E_o - E_s$$

$$= -\frac{GMm}{2(7R)} - \left[-\frac{GMm}{R}\right] \quad (\because r = 7R)$$

$$= -\frac{GMm}{14R} + \frac{GMm}{R} = \frac{13}{14}\frac{GMm}{R}$$

21. (b)

22. (b): According to equation of continuity,

$$a_1 v_1 = a_2 v_2$$

$$\therefore \quad (40 \times 10^{-8}) \times v_1 = 8 \times 10^{-4} \times \left(\frac{0.15}{60}\right)$$

$$\text{or} \quad v_1 = \frac{8 \times 10^{-4} \times 0.15}{40 \times 10^{-8} \times 60} = 5 \text{ m s}^{-1}$$

23. (b): Let h be maximum height reached by the body. Taking motion of the body from half the maximum height upto the highest point, we have

$u = 10$ m s^{-1}, $a = -g = -10$ m s^{-2}, $v = 0$, $S = \frac{h}{2}$

As $v^2 = u^2 + 2aS$

$$\therefore \quad 0 = 10^2 + 2(-10) \times \frac{h}{2} \quad \text{or } h = 10 \text{ m}$$

24. (a): In the given reaction,

$$\text{B.E.} = [(m({}^6_3\text{Li}) + m({}^2_1\text{H}) - 2m({}^4_2\text{He})] \times 931 = 22.4$$

$$\therefore \quad m({}^6_3\text{Li}) = \frac{22.4}{931} - 2.0140 + 2 \times 4.0026$$

$$= 6.015 \text{ amu}$$

25. (a): Range, $R = \frac{u^2 \sin 2\theta}{g}$

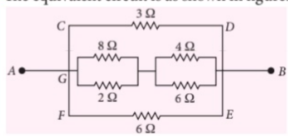
For maximum range, angle of projection θ is 45°.

$$\therefore \quad R_{max} = \frac{u^2 \sin(2 \times 45°)}{g} = \frac{u^2}{g} = 1000 \text{ m} \quad \ldots\text{(i)}$$

$$\therefore \quad \text{Maximum height, } H = \frac{u^2 \sin^2 45°}{2g}$$

$$H = \frac{1}{2} \times 1000 \times \left(\frac{1}{\sqrt{2}}\right)^2 = 250 \text{ m} \quad \text{(Using (i))}$$

26. (b): The equivalent circuit is as shown in figure.



Resistance of arm AGB

$$= \frac{8 \times 2}{8 + 2} + \frac{4 \times 6}{4 + 6} = 1.6 + 2.4 = 4\ \Omega$$

For equivalent resistance between A and B, the resistance of arms CD, AGB and FE are in parallel. Thus

$$\frac{1}{R_{AB}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4+3+2}{12} = \frac{9}{12}$$

$$\text{or} \quad R_{AB} = \frac{12}{9} = \frac{4}{3}\ \Omega$$

27. (c): According to Kepler's third law $T^2 \propto R^3$

$$T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 4\left(\frac{3R}{R}\right)^{3/2} = 4\sqrt{27} \text{ hours}$$

28. (c): For SHM,

Maximum velocity, $v_{max} = A\omega$

$$\text{For 1}^{\text{st}} \text{ case, } v_{max_1} = A_1\omega_1 = A_1\sqrt{\frac{k_1}{M}} \quad \ldots\text{(i)}$$

$$\text{For II}^{\text{nd}} \text{ case, } v_{max_2} = A_2\omega_2 = A_2\sqrt{\frac{k_2}{M}} \quad \ldots\text{(ii)}$$

According to question,

$$v_{max_1} = v_{max_2}$$

$$\therefore \quad A_1\sqrt{\frac{k_1}{M}} = A_2\sqrt{\frac{k_2}{M}} \quad \text{or} \quad \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

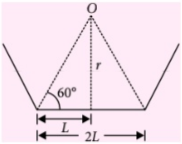
29. (a): Length of each side of hexagon = $2L$

Mass of each side = m

Let O be centre of mass of hexagon.

Therefore, perpendicular distance of O from each side

$r = L\tan 60° = L\sqrt{3}$



The desired moment of inertia of hexagon about O is

$I = 6\,[I_{\text{one side}}]$

$= 6\left[\frac{m(2L)^2}{12} + mr^2\right]$

$= 6\left[\frac{mL^2}{3} + m(L\sqrt{3})^2\right] = 6\left[\frac{mL^2}{3} + 3mL^2\right] = 20\,mL^2$

30. (c)

31. (c) : According to Gauss's law, the total outward electric flux linked with gaussian surface

$\phi_E = \frac{1}{\varepsilon_0} \times$ charge enclosed by surface.

If the radius of the gaussian surface is doubled the total outward electric flux will remain the same as charge enclosed by the guassian surface is unchanged.

32. (c) : Here, P = 100 W, r = 1.0 m

Intensity of radiation at distance r, $I = \frac{P}{4\pi r^2}$

Let maximum energy density in the electric field $= u_{E_{max}}$

Then $I = u_{E_{max}} c = \frac{P}{4\pi r^2}$

or $\quad u_{E_{max}} = \frac{P}{4\pi r^2 c}$

$= \frac{100}{4\times 3.14\times(1)^2\times(3\times 10^8)} = 2.65 \times 10^{-8}\ \text{J m}^{-3}$

33. (c) : When one slit is closed, amplitude becomes half and intensity becomes $1/4^{th}$

i.e., $I_0 = \frac{1}{4}I$ or $I = 4I_0$

34. (c) : In L–R circuit, current at any time t is given by

$I = \frac{V}{R}\left[1 - e^{\frac{-R}{L}t}\right] = \frac{V}{R} - \frac{V}{R}e^{\frac{-R}{L}t}$ ...(i)

$\frac{dI}{dt} = \frac{V}{R}e^{\frac{-R}{L}t}\left(\frac{R}{L}\right) = \frac{V}{L}e^{\frac{-R}{L}t}$

Induced emf $\varepsilon = L\frac{dI}{dt} = Ve^{\frac{-R}{L}t}$ ...(ii)

From (i), $IR = V - Ve^{\frac{-R}{L}t}$

Using (ii), $IR = V - \varepsilon$ or $\varepsilon = V - IR$

Therefore, graph between ε and I is a straight line with negative slope and positive intercept.

35. (c) : $t_1 = 3 = 2\pi\sqrt{\frac{I}{MR}}$. ...(i)

where R is resultant intensity of earth's field

$t_2 = 3\sqrt{2} = 2\pi\sqrt{\frac{I}{MH}}$ ...(ii)

Dividing eqn.(i) by eqn.(ii)

$\frac{1}{\sqrt{2}} = \sqrt{\frac{H}{R}} = \sqrt{\frac{R\cos\delta}{R}} = \sqrt{\cos\delta}$

$\cos\delta = \frac{1}{2}, \delta = 60°$

36. (b): Resistance between Q and S, $R' = \frac{5\times 10}{5+10} = \frac{10}{3}\,\Omega$

Potential difference across Q and S,

$V_Q - V_S = \frac{2\times 10}{3} = \frac{20}{3}$ V

Current through arm QPS, $I_1 = \frac{20}{3\times 5} = \frac{4}{3}$ A

$V_Q - V_P = \frac{4}{3}\times 2 = \frac{8}{3}$ V

Current through arm QRS, $I_2 = \frac{20/3}{10} = \frac{2}{3}$ A

$V_Q - V_R = \frac{2}{3}\times 3 = 2$ V

$V_P - V_R = (V_Q - V_R) - (V_Q - V_P)$

$= 2 - \frac{8}{3} = \frac{-2}{3} \approx -1$ V.

37. (b): Reverse resistance $= \frac{\Delta V}{\Delta I} = \frac{15-10}{(35-10)\times 10^{-6}}$

$= 200 \times 10^3\ \Omega = 200$ kΩ

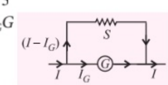
38. (c) : Due to change in temperature t°C, increase in length,

$\Delta l = l\alpha t$ or $\frac{\Delta l}{l} = \alpha t$

$Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\Delta l / l}$ $\therefore$ Stress $= Y \times \frac{\Delta l}{l} = Y\alpha t$

39. (d): In case of a galvanometer, $I \propto \theta$

So, $\frac{I_G}{I} = \frac{10}{50} = \frac{1}{5}$ *i.e.*, $I_G = \frac{1}{5}I$



From figure, $(I - I_G)S = I_G G$

i.e., $(I - \frac{1}{5}I)\times 12 = \frac{1}{5}IG$

or, $G = 4 \times 12 = 48\ \Omega$

40. (b): $\frac{3h}{2\pi} = n\left(\frac{h}{2\pi}\right)$ $\therefore$ $n = 3$

The kinetic energy of the electron in n^{th} orbit is

$K_n = \frac{13.6}{n^2}$ eV

$\therefore$ $K_3 = \frac{13.6}{3^2}$ eV $= \frac{13.6}{9}$ eV $= 1.51$ eV

◈◈

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 120

Units and Measurements

Time Taken : 60 Min.

NEET / PMTs

Only One Option Correct Type

1. Which of the following is the most accurate measurement ?
   (a) 20×10^{-3} m (b) 200×10^{-4} m
   (c) 2×10^{-2} m (d) 0.02 m

2. When a current of (2.5 ± 0.5) A flows through a wire, it developes a potential difference of (20 ± 1)V. The resistance of wire is
   (a) $(8 \pm 2)\ \Omega$ (b) $(8 \pm 1.6)\ \Omega$
   (c) $(8 \pm 1.5)\ \Omega$ (d) $(8 \pm 3)\ \Omega$

3. $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$. The value of n is
   (a) 0 (b) −1
   (c) 1 (d) None of these

4. Crane is British unit of volume (one crane = 170.4742 L). Convert crane into SI units.
   (a) $0.170474\ m^3$ (b) $17.0474\ m^3$
   (c) $0.00170474\ m^3$ (d) $1704.74\ m^3$

5. The radius of the proton is about 10^{-15}m. The radius of the observable universe is 10^{26} m. Identify the distance which is half-way between, these two extremes on a logarithmic scale.
   (a) 10^{21} m (b) 10^{6} m (c) 10^{-6} m (d) 10^{0} m

6. The pitch of a screw guage is 1 mm and there are 100 divisions on the circular scale. While measuring diameter of a thick wire, the pitch scale reads 1 mm and 63rd division on the circular scale coincides with the reference. The length of the wire is 5.6 cm. Then
   (a) The least count of screw guage is 0.001 mm
   (b) The volume of the wire is $0.117\ cm^3$
   (c) The diameter of the wire is 1.63 m
   (d) The cross-section area of the wire is $0.0209\ cm^3$

7. There are atomic clocks capable of measuring time with an accuracy of 1 part in 10^{11}. If two such clocks are operated with precision, then after running for 5000 yr, these will record a difference of
   (a) nearly 2 s (b) 1 day
   (c) 10^{11} s (d) 1 yr

8. SI unit of intensity of wave is
   (a) $J\ m^{-2}s^{-2}$ (b) $J\ m^{-1}s^{-2}$
   (c) $W\ m^{-2}$ (d) $J\ m^{-2}$

9. The equation of stationary wave is $y = A\sin kx \cos \omega t$, where, y and x are in metre and t in second. Choose the correct option.
   (a) The dimensions of A and k are same
   (b) The dimensions of A, k and ω are same
   (c) The dimensions of k and ω are same
   (d) The dimensions of (kx) and (ωt) are same

10. A container contains 35 kg water. 0.2 kg water leaks from the container. Find the amount of water in container.
    (a) 34.8 kg (b) 35 kg
    (c) 34.80 kg (d) 35.0 kg

11. In the relation $p = \frac{\alpha}{\beta} e^{-\frac{\alpha Z}{k\theta}}$, where p is pressure, Z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be
    (a) $[M^0L^2T^0]$ (b) $[ML^2T]$
    (c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

12. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. The thickness of hair is
(a) 0.035 mm (b) 0.04 mm
(c) 0.35 mm (d) 0.40 mm

Assertion & Reason Type

Directions : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false.

13. **Assertion :** A dimensionally correct equation may or may not be correct.

Reason : A dimensionally incorrect equation must be wrong.

14. **Assertion :** If two quantities have same dimensions, then they must represent same physical quantities.

Reason : Work and torque have different dimensions.

15. **Assertion :** Power of a engine depends on mass, angular speed, torque and angular momentum, then the formula of power is not derived with the help of dimensional method.

Reason : In mechanics, if a particular quantity depends on more than three quantities, then we can not derive the formula of the quantity by the help of dimensional method.

JEE MAIN / JEE ADVANCED / PETs

Only One Option Correct Type

16. A gas bubble from an explosion under water oscillates with a period T proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion. The values of a, b and c are
(a) $a = 0, b = 1, c = 2$ (b) $a = 1, b = 2, c = 3$
(c) $a=\frac{5}{6}, b=\frac{1}{2}, c=\frac{1}{3}$ (d) $a=\frac{-5}{6}, b=\frac{1}{2}, c=\frac{1}{3}$

17. A system has basic dimensions as density $[D]$, velocity $[V]$ and area $[A]$. The dimensional representation of force in this system is
(a) $[AV^2D]$ (b) $[A^2VD]$
(c) $[AVD^2]$ (d) $[A^0VD]$

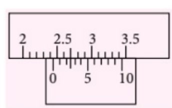
18. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°), then the least count of the instrument is
(a) one minute (b) half minute
(c) one degree (d) half degree

19. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of 0.01 mm. Take $g = 9.8$ m s^{-2} (exact). The Young's modulus obtained from the reading is
(a) $(2.0 \pm 0.3)\ 10^{11}$ N m^{-2}
(b) $(2.0 \pm 0.2) \times 10^{11}$ N m^{-2}
(c) $(2.0 \pm 0.1) \times 10^{11}$ N m^{-2}
(d) $(2.0 \pm 0.05) \times 10^{11}$ N m^{-2}

More than One Options Correct Type

20. If p, ρ, g and h denote pressure, density, acceleration due to gravity and height, respectively. In the case of flow of fluid, $\frac{p}{\rho g}+\frac{v^2}{2g}+h=u_0$ where v denotes velocity.
Mark the correct options.
(a) The equation is dimensionally correct.
(b) If dimensions of u_0 are $[M^0LT^0]$, the equation is dimensionally correct.
(c) Dimensions of $\frac{p}{\rho g}$ are same as that of $\frac{v^2}{2g}$.
(d) None of the above.

21. In the given figure, main scale is graduated in millimeter. Mark the correct options, if least count is 0.1 mm.



(a) The main scale reading is 2.4 cm.
(b) The vernier scale reading is 0.3 mm.
(c) The total reading is 2.43 cm.
(d) The total reading is 2.4 cm.

22. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stopwatch with the least count of

1 s for this and records 40 s for 20 oscillations. For the observation, which of the following statement(s) is/are correct?

(a) Error ΔT in measuring T, the time period is 0.05 s.
(b) Error ΔT in measuring T, the time period is 1 s.
(c) Percentage error in the determination of g is 5%.
(d) Percentage error in the determination of g is 2.5%.

23. If $\left(A+\frac{B}{C^2}\right)(D-x)=y$, then

(a) dimensions of A and B must be same
(b) dimensions of A and B may be same
(c) dimensions of D and x must be same
(d) dimensions of AD and y must be same

Numerical Value Type

24. The time period of oscillation of a body is given by

$$T=2\pi\sqrt{\frac{mgA}{K}}$$

K : Represents the kinetic energy, m mass, g acceleration due to gravity and A is unknown.
If $[A]=M^x L^y T^z$
Then what is the value of $x+y+z$?

25. If $P=\frac{x^3y^3}{z\sqrt{w}}$, the percentage error in x, y, z and w are 1%, 1%, 1% and 4% respectively. Find percentage error in P.

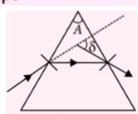
26. The plate current in a triod can be written as

$$I=K\left(V_g+\frac{V_p}{\mu}\right)^{3/2}.$$

Here, V_g and V_p denote potential. In the dimensional formula of K, find the ratio of dimensions of length and mass.

Comprehension Type

The refractive index μ of the material of glass prism is given by $\mu=\frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$



Glass prism

where, A = Angle of prism, δ_{min} = Angle of minimum deviation.

A glass prism of 60° gives angle of minimum deviation as 36° with the maximum error of 1.05° when a beam of parallel light passed through the prism during experiment. (Given, cot 48° = 0.900404 and sin 48° = 0.743).

27. Find the percentage error in the measurement of refractive index of the material of the prism.

(a) 1% (b) 1.65%
(c) 4% (d) 3.3%

28. Find the range of experimental value of refractive index μ.

(a) $1.46 \le \mu \le 1.51$ (b) $2 \ge \mu \ge 1.51$
(c) $0 < \mu < \infty$ (d) $1.46 \le \mu \le 2.51$

Matrix Match Type

29.

Column I	Column II
(A) Inductance	(P) $[K^{-1}ML^2T^{-2}]$
(B) Electric resistivity	(Q) $[M^{-1}T^{-1}]$
(C) Specific activity	(R) $[A^{-2}ML^3T^{-3}]$
(D) Boltzmann's constant	(S) $[A^{-2}ML^2T^{-2}]$

	A	B	C	D
(a)	P	Q	R	S
(b)	Q	P	S	R
(c)	R	S	P	Q
(d)	S	R	Q	P

30.

Column I	Column II
(A) Same dimension in mass	(P) electric field
(B) same dimension in time	(Q) magnetic field
(C) (–1) dimension in current	(R) inductive reactance
(D) zero dimension in length	(S) resistivity

	A	B	C	D
(a)	P,S	P,Q,R	R,S	P
(b)	P,Q,R	P,R	R,S	Q
(c)	P,Q,R,S	P,Q,R	P,Q	Q
(d)	P,Q,R,S	P,R	P,Q	P

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SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 120

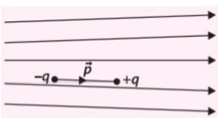
Electric Charges and Fields

Time Taken : 60 Min.

NEET / PMTs

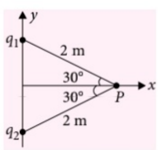
Only One Option Correct Type

1. Electric field lines in which an electric dipole is placed are as shown in the figure. Which of the following statements is correct ?



(a) The dipole will not experience any force.
(b) The dipole will experience a force towards right.
(c) The dipole will experience a force towards left.
(d) The dipole will experience a force upwards.

2. Two point charges $q_1 = -4\ \mu C$ and $q_2 = 8\ \mu C$ are lying on the y-axis. They are equidistant from the point P, which lies on the x-axis. A small object of charge $q_0 = 8\ \mu C$ and mass $m = 12$ g is placed at P. When it is released what is its acceleration in m s^{-2} ?



(a) $3\sqrt{3}\hat{i}+9\hat{j}$ (b) $9\hat{i}+3\sqrt{3}\hat{j}$
(c) $3\hat{i}+3\sqrt{3}\hat{j}$ (d) $3\sqrt{3}\hat{i}+3\hat{j}$

3. What total charge q must a disk of radius 2.50 cm carry so that the electric field on the surface of the disk at its center equals the value at which air breaks down electrically, producing sparks ?
(Electrical break down of air = 3×10^6 N C^{-1})
(a) 208 nC (b) 156 nC
(c) 52 nC (d) 104 nC

4. A proton orbits with a speed v = 294 km s^{-1} just outside a charged sphere of radius r = 1.13 cm. What is the charge on the sphere ?
(a) −1.13 nC (b) +1.13 nC
(c) −2.26 nC (d) + 2.26 nC

5. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. What is the maximum flux of the electric field through the cube ?
(a) $\frac{a\lambda}{\varepsilon_0}$ (b) $\frac{\sqrt{2}\,a\lambda}{\varepsilon_0}$ (c) $\frac{\sqrt{3}\,a\lambda}{\varepsilon_0}$ (d) $\frac{2a\lambda}{\varepsilon_0}$

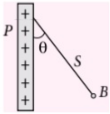
6. A charged particle of mass 5×10^{-6} g is kept over a large horizontal sheet of charge density 4×10^{-6} C m^{-2}. How many electrons should be removed to give some charge to the particle so that if released it does not fall down ?
(a) 1.36×10^6 (b) 2.72×10^6
(c) 1.6×10^7 (d) 3.2×10^6

7. A and B are two identical spherical charged bodies which repel each other with force F, kept at a finite distance. A third uncharged sphere of the same size is brought in contact with sphere B and removed. It is then kept at midpoint of A and B. The magnitude of force on C is
(a) $F/2$ (b) $F/8$ (c) F (d) zero

8. An electric dipole is placed at an angle of 60° with an electric field of intensity 10^5 N C^{-1}. It experiences a torque equal to $8\sqrt{3}$ N m. What is the charge on the dipole, if dipole length is 2 cm ?

(a) 16×10^{-3}C (b) 32×10^{-3}C
(c) 4×10^{-3}C (d) 8×10^{-3}C

9. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P as shown in the figure. The surface charge density of the sheet is proportional to



(a) $\cos\theta$ (b) $\cot\theta$ (c) $\sin\theta$ (d) $\tan\theta$

10. An imaginary, closed, spherical surface S of radius R is centered on the origin. A positive charge $+q$ is originally at the origin, and the flux through the surface is Φ_E. Three additional charges are now added along the x-axis : $-3q$ at $x = -R/2$, $+5q$ at $x = R/2$, and $+4q$ at $x = 3R/2$. The flux through S is now

(a) $2\Phi_E$ (b) $3\Phi_E$ (c) $6\Phi_E$ (d) $7\Phi_E$

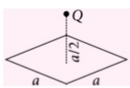
11. Three conducting metal spherical shells of radii R, $2R$ and $3R$ are given charges Q_1, Q_2 and Q_3 respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. The ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$ is

(a) 1 : 2 : 3 (b) 1 : 3 : 5
(c) 1 : 4 : 9 (d) 1 : 8 : 18

12. A charge Q is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge a as shown in the figure. Find the flux of the electric field through the square surface.

(a) $\dfrac{Q}{\varepsilon_0}$ (b) $\dfrac{Q}{2\varepsilon_0}$

(c) $\dfrac{Q}{4\varepsilon_0}$ (d) $\dfrac{Q}{6\varepsilon_0}$



Assertion & Reason Type

Directions : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion
(b) If both assertion and reason are true but reason is not the correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false.

13. Assertion : In electrostatics, electric lines of force can never be closed loops, as a line can never start and end on the same charge.

Reason : The number of electric lines of force originating or terminating on a charge is proportional to the magnitude of charge.

14. Assertion : If a point charge q is placed in front of an infinite grounded conducting plane surface, the point charge will experience a force.

Reason : This force is due to the induced charge on the conducting surface which is at zero potential.

15. Assertion : Charge is quantized.

Reason : Charge which is less than 1 C is not possible.

JEE MAIN / JEE ADVANCED / PETs

Only One Option Correct Type

16. In a 1911 paper, Ernest Rutherford said : In order to form some idea of the forces required to deflect an alpha particle through a large angle, consider an atom containing a point positive charge Ze at its center and surrounded by a distribution of negative electricity, $-Ze$ uniformly distributed within a sphere of radius R. The electric field E at a distance r from the center for a point inside the atom is

(a) $\dfrac{Ze}{4\pi\varepsilon_0}\left(\dfrac{1}{r^2} - \dfrac{r}{R^3}\right)$ (b) $\dfrac{Ze}{4\pi\varepsilon_0 r^2}$

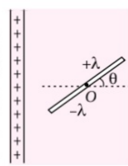
(c) $\dfrac{Zer}{4\pi\varepsilon_0 R^2}$ (d) $\dfrac{Ze}{4\pi\varepsilon_0}\left(\dfrac{1}{r^2} + \dfrac{r}{R^3}\right)$

17. Electric field on the axis of a small electric dipole at a distance r is $\vec{E}_1$ and $\vec{E}_2$ at a distance of $2r$ on a line of perpendicular bisector. Then

(a) $\vec{E}_2 = \dfrac{-\vec{E}_1}{8}$ (b) $\vec{E}_2 = \dfrac{-\vec{E}_1}{16}$

(c) $\vec{E}_2 = \dfrac{-\vec{E}_1}{4}$ (d) $\vec{E}_2 = \dfrac{-\vec{E}_1}{8}$

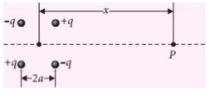
18. A large sheet carries uniform surface charge density σ. A rod of length $2l$ has a linear charge density λ on one half and $-\lambda$ on the second half. The rod is hinged at mid point O and makes an angle θ with the normal to the sheet. The torque experienced by the rod is



(a) zero (b) $\dfrac{\sigma\lambda l^2}{2\varepsilon_0}\sin\theta$

(c) $\dfrac{\sigma\lambda l^2}{\varepsilon_0}\sin\theta$ (d) $\dfrac{\sigma\lambda l^2}{2\varepsilon_0}$

19. One type of electric quadrupole is formed by four charges located at the vertices of a square of side $2a$. Point P lies a distance x from the center of the quadrupole on a line parallel to two sides of the square as shown in the figure. For $x >> a$, the electric field at P is approximately equal to



(a) $\frac{3(2qa^2)}{2\pi\varepsilon_0 x^4}$ (b) $\frac{4(2qa^2)}{2\pi\varepsilon_0 x^4}$

(c) $\frac{3(2qa^2)}{4\pi\varepsilon_0 x^4}$ (d) $\frac{3(2qa)}{2\pi\varepsilon_0 x^3}$

More than One Options Correct Type

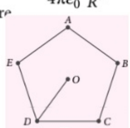
20. Measured values of the electric field E at a distance z along the axis of a charged plastic disk are given here :

z (cm)	E (10^7 N C^{-1})
0	2.043
1	1.732
2	1.442
3	1.187
4	0.972
5	0.797

(a) The radius of the disk is 6.5 cm.
(b) The radius of the disk is 13 cm.
(c) The charge on the disk is 4.8 μC.
(d) The charge on the disk is 9.8 μC.

21. A solid non-conducting sphere of radius R carries a non-uniform charge distribution, with charge density $\rho = \rho_s r/R$, where ρ_s is a constant and r is the distance from the center of the sphere.
(a) The total charge, Q on the sphere is $\pi\rho_s R^3$.
(b) The total charge, Q on the sphere is $\frac{4}{3}\pi\rho_s R^3$.
(c) The electric field inside the sphere is $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^4}r^2$.
(d) The electric field inside the sphere $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$.

22. Five charges, q each are placedat the corners of a regular pentagon $ABCDE$ of side a as shown in the figure.



(a) The electric field at the centre of the pentagon O will be zero.
(b) The electric field at O will be $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ if the charge from one of the corners (say A) is removed.
(c) The electric field at O will be $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ if the charge q at A is replaced by $-q$.
(d) The electric field at O will be $\frac{1}{4\pi\varepsilon_0}\cdot\frac{nq}{a^2}$ if pentagon is replaced by n-sided regular polygon with charge q at each of its corners.

23. A particle of charge q and mass m moves rectilinearly under the action of a electric field $E = A - Bx$ where B is a positive constant with appropriate unit and x is distance from the point where the particle was initially at rest.
(a) The distance travelled by the particle till it comes to rest is $2A/B$.
(b) Acceleration of the particle at the instant before coming to rest is $\frac{-qA}{m}$.
(c) The speed of the particle as a function of x is $\sqrt{\frac{2q}{m}\left(Ax - \frac{Bx^2}{2}\right)}$.
(d) The speed of the particle as a function of x is $\sqrt{\frac{2}{m}(Ax - Bx^2)}$.

Numerical Value Type

24. A point charge 0.140 nC is placed on the apex of a cone of semi-vertical angle 30°. Find the electric flux (in SI units) through the base of cone.

25. Point charges, q_1 and q_2, are placed on the X-axis, with q_1, at $x = 0$ and q_2 at $x = d$. A third point charge $+Q$, is placed at $x = 3d/4$. If the net electrostatic force experienced by the charge $+Q$ is zero, then find the ratio q_1/q_2.

26. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k(q^2/\gamma)^{1/N}$, where k is a constant. Then find N.

Comprehension Type

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

27. Taking the electronic charge as e and the permitivity as ε_0, use dimensional analysis to determine the correct expression for ω_p.

(a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

28. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27} m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm
(c) 300 nm (d) 200 nm

Matrix Match Type

29. Column I gives the dependence of electric field (E) on distance (r) due to certain charged objects and Column II lists these objects. Match the entries of Column I with the entries of Column II.

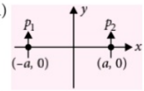
Column I	Column II
(A) r^0	(P) Point charge
(B) r^{-1}	(Q) Thin infinitely long wire of uniform linear charge density
(C) r^{-3}	(R) Infinite uniformly charged plane sheet
(D) r^{-2}	(S) Electric field inside the region between two uniformly charged parallel planes
	(T) Electric field on the axial line of a short electric dipole.

Codes

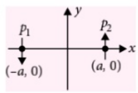
	A	B	C	D
(a)	P	Q	R	S, T
(b)	R, S	Q	T	P
(c)	R, S	P, Q	T	P
(d)	P, T	R	Q	S

30. In each situation of column I two electric point dipoles having dipole moments $\vec{p}_1$ and $\vec{p}_2$ of same magnitude (that is, $p_1 = p_2$) are placed on x-axis symmetrically about origin in different orientations as shown. In column II certain inferences are drawn for these two dipoles. Match the different orientations of dipoles in column I with the corresponding results in column II.

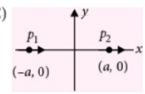
Column I / **Column II**

(A)


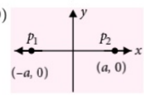
(P) The torque on one dipole due to other is zero

(B)


(Q) The potential energy of one dipole in electric field of other dipole is negative

(C)


(R) There is one straight line in x-y plane (not at infinity) which is equipotential

(D)


(S) Electric field at origin is zero

Codes

	A	B	C	D
(a)	P, R	P, Q, R, S	P, Q, S	P
(b)	Q, R	P, R	P, R	Q, S
(c)	P, Q, S	P, R, S	P, R	Q, S
(d)	R, S	P, R	P, S	P, Q

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